

Faire de la physique avec *Interstellar*



Roland Lehoucq, CEA-Saclay Service d'Astrophysique

FROM CHRISTOPHER NOLAN

INTERSTELLAR TIMELINE

PLAN A

Use data gathered from the gravitational anomalies to solve the quantum gravity equations. Use the solution to develop gravity-manipulating technology and construct ships to evacuate those on Earth.

PLAN B

Start a human colony offworld using a small group of settlers and a massive zygote bank to ensure genetic diversity. Those left on Earth will perish as a result of the blight.

Character Key

Cooper Murph Amelia Romilly Doyle
TARS Prof. Brand Mann Miller Edmund

THE TESSERACT

Time Dilation Factor: N/A

A hyperdimensional realm beyond spacetime as we know it, where time in the Universe is a tangible spatial dimension. Created by the descendants of humanity in the far future as part of a stable temporal loop.



EARTH

Time Dilation Factor: 0

The home of humanity, ravaged by a blight that is slowly terraforming the atmosphere.

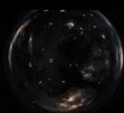


The Lazarus Missions: 12 of the bravest humans are sent to 12 different potentially habitable worlds on the other side of the wormhole. Each is equipped with a robot, a stasis pod, and 2 years of life support. Communication with Earth will be limited; they'll be able to send back a "thumbs up" or "thumbs down" indicating whether or not their world is viable for long-term habitation. They are aware it is most likely a suicide mission.

THE WORMHOLE

Time Dilation Factor: Negligible

A region of intensely curved spacetime connecting the Solar System with Gargantua's system. Created by the descendants of humanity in the far future as part of a stable temporal loop.



MANN'S WORLD

Time Dilation Factor: Negligible

An icy planet covered in a sponge-like array of frozen clouds and glaciers that reach for the sky.



EDMUND'S WORLD

Time Dilation Factor: Negligible

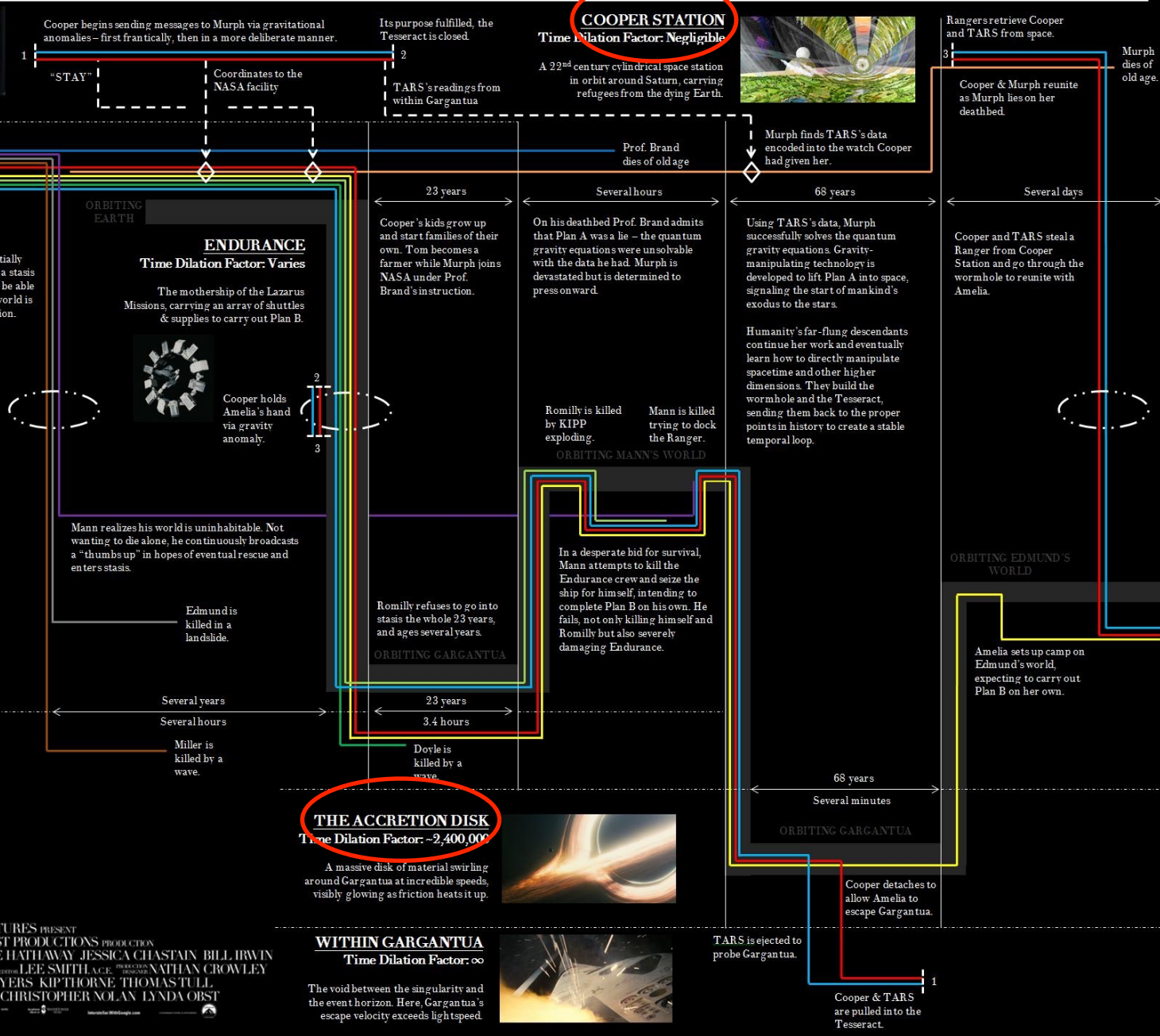
A rocky wasteland with a breathable atmosphere and comfortable gravity.



MILLER'S WORLD

Time Dilation Factor: -61,000

A high-gravity ocean world continuously swept by skyscraper-size waves.



THE ACCRETION DISK

Time Dilation Factor: ~2,400,000

A massive disk of material swirling around Gargantua at incredible speeds, visibly glowing as friction heats it up.



WITHIN GARGANTUA

Time Dilation Factor: ∞

The void between the singularity and the event horizon. Here, Gargantua's escape velocity exceeds lightspeed.



Timeline Version 1 by stoifcs42 from NolanFans (sto-ifcs42 on Reddit).

All images are from the film, except for Cooper Station (painting of an O'Neill cylinder) and Edmund's world (picture of Meteor Crater taken by stoifcs42).

Film credits:

PARAMOUNT PICTURES AND WARNER BROS. PICTURES PRESENT
 IN ASSOCIATION WITH LEGENDARY PICTURES A SYNCOPY/LYNDY OBST PRODUCTIONS PRODUCTION
 A FILM BY CHRISTOPHER NOLAN "INTERSTELLAR" MATTHEW MCCONAUGHEY ANNE HATHAWAY JESSICA CHASTAIN BILL IRWIN
 ELLEN BURSTYN AND MICHAEL CAINE COSTUME DESIGNER MARY ZOPHRES MUSIC BY HANS ZIMMER EDITOR LEE SMITH ACE PRODUCTION DESIGNER NATHAN CROWLEY
 DIRECTOR OF PHOTOGRAPHY HOYTEVAN HOYTEMA, A.S.C. EXECUTIVE PRODUCERS JORDAN GOLDBERG JAKE MYERS KIP THORNE THOMAS TULL
 PRODUCED BY JONATHAN NOLAN AND CHRISTOPHER NOLAN PRODUCED BY EMMA THOMAS CHRISTOPHER NOLAN LYNDY OBST
 WRITTEN BY CHRISTOPHER NOLAN DIRECTED BY CHRISTOPHER NOLAN
 PG-13 PARENTS STRONGLY CAUTIONED
 www.paramount.com www.warnerbros.com www.legendary.com www.syncopy.com www.lyndyobst.com www.interstellar.com

La gravitation et le temps
sont au cœur d'*Interstellar*

Gargantua, son disque d'accrétion et la planète de Miller.

Universalité de la chute libre

BBC TWO

masse inertielle = masse gravitationnelle

... une idée [la plus heureuse de ma vie] me vint :

une personne en chute libre ne sentira plus son poids.

Je restai ébahi. Cette pensée d'apparence si simple me fit une grande impression. Elle me poussa vers une nouvelle théorie de la gravitation.

Albert Einstein (1907)

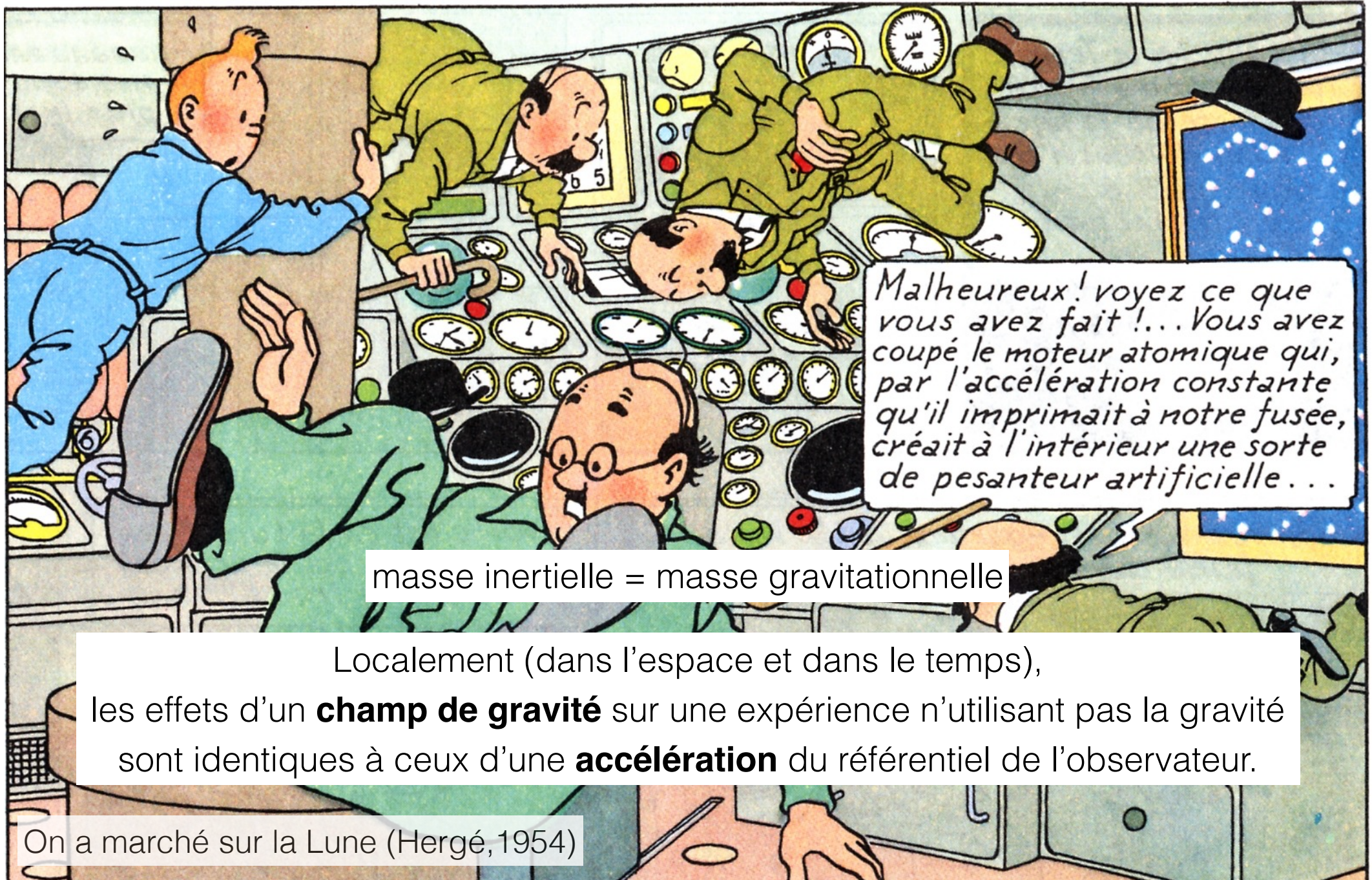
Extrait de la vidéo « World's biggest vacuum chamber »

<https://www.youtube.com/watch?v=E43-CfukEgs>



22 secondes de microgravité dans l'Airbus A300 « Zero-G » de Novespace.

Principe d'équivalence d'Einstein



Malheureux! voyez ce que vous avez fait!... Vous avez coupé le moteur atomique qui, par l'accélération constante qu'il imprimait à notre fusée, créait à l'intérieur une sorte de pesanteur artificielle...

masse inertielle = masse gravitationnelle

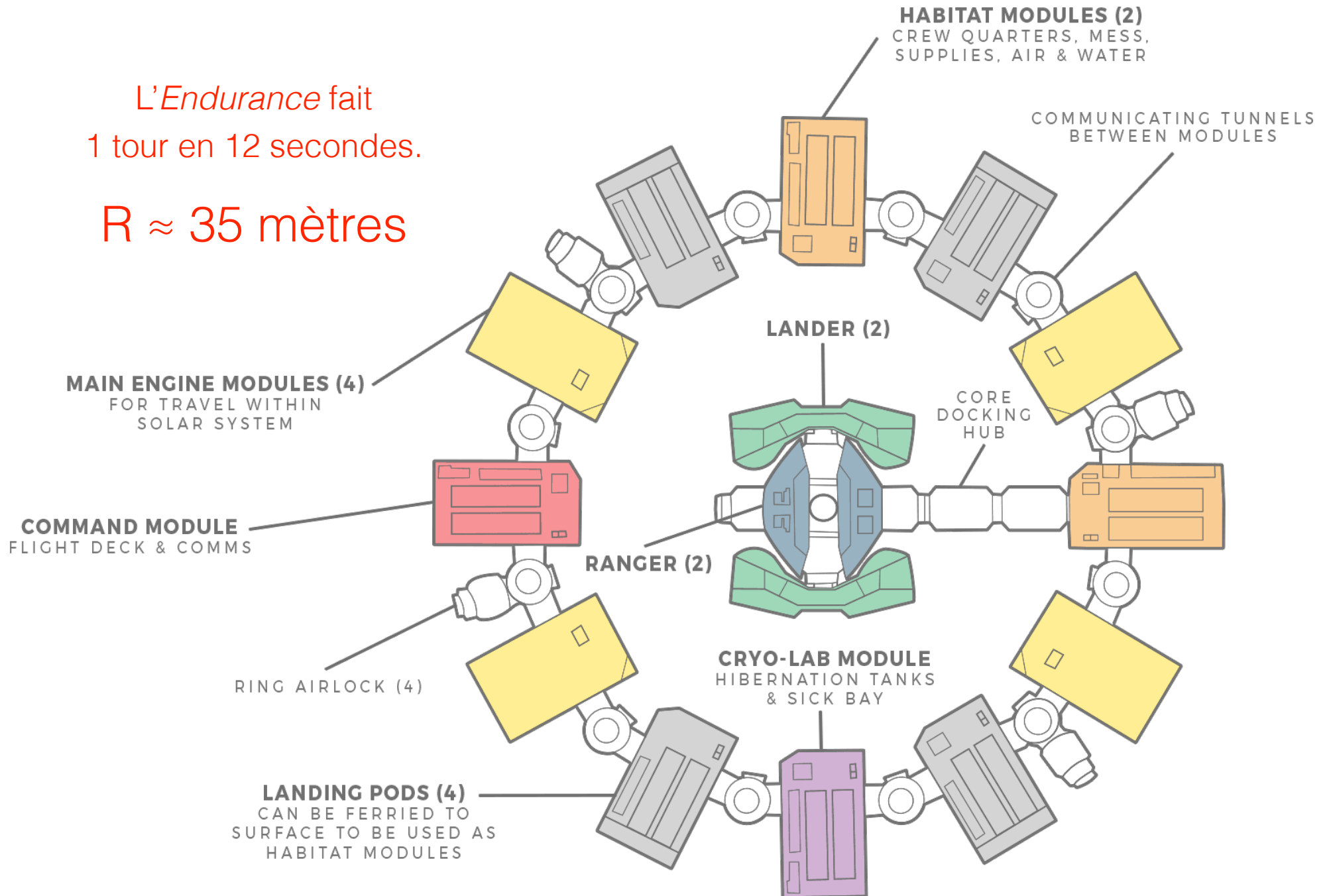
Localement (dans l'espace et dans le temps), les effets d'un **champ de gravité** sur une expérience n'utilisant pas la gravité sont identiques à ceux d'une **accélération** du référentiel de l'observateur.

On a marché sur la Lune (Hergé, 1954)

Le vaisseau *Endurance*

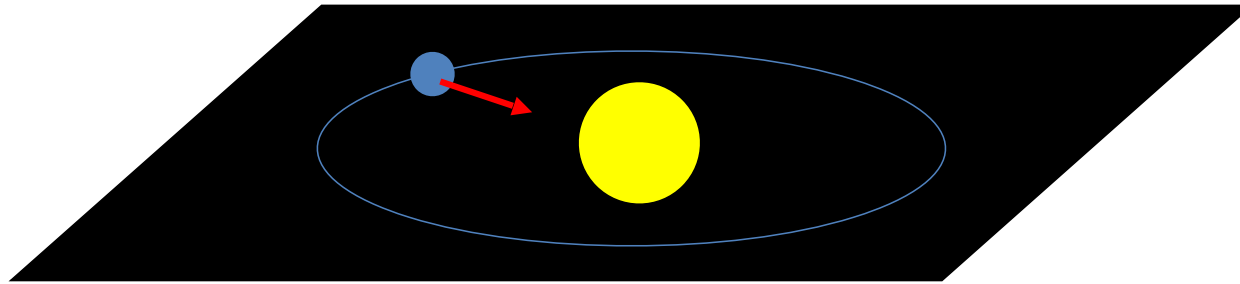
L'*Endurance* fait
1 tour en 12 secondes.

$R \approx 35$ mètres

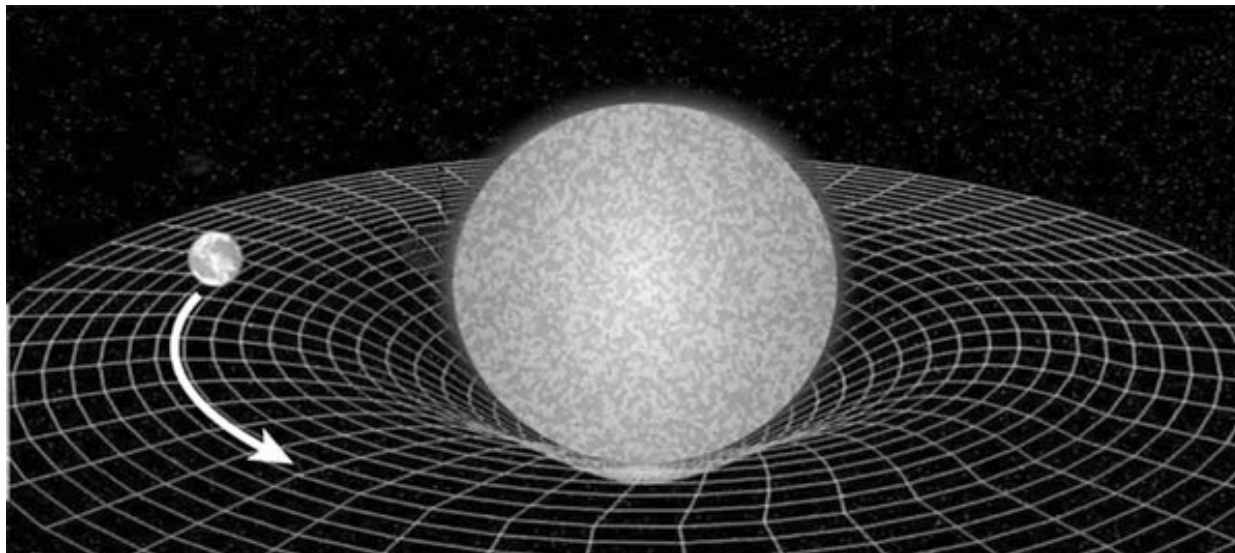


Nouvelle vision de la gravité

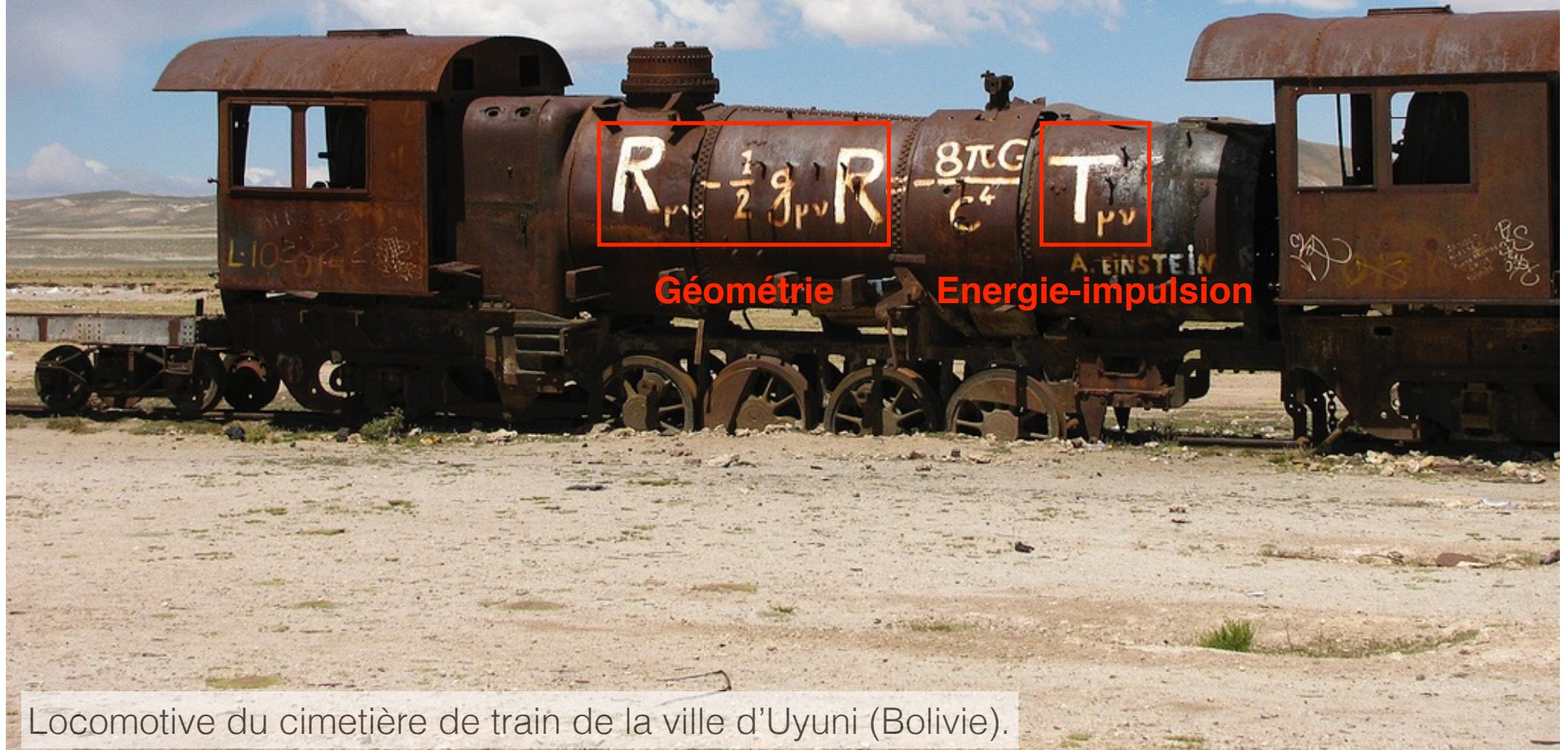
Pour **Newton** la gravité est une force qui agit instantanément entre corps massifs.



Pour **Einstein**, la gravité est une manifestation de la courbure de l'espace-temps imposée par la distribution & flux de matière et d'énergie.



Nouvelles équations !



Locomotive du cimetière de train de la ville d'Uyuni (Bolivie).

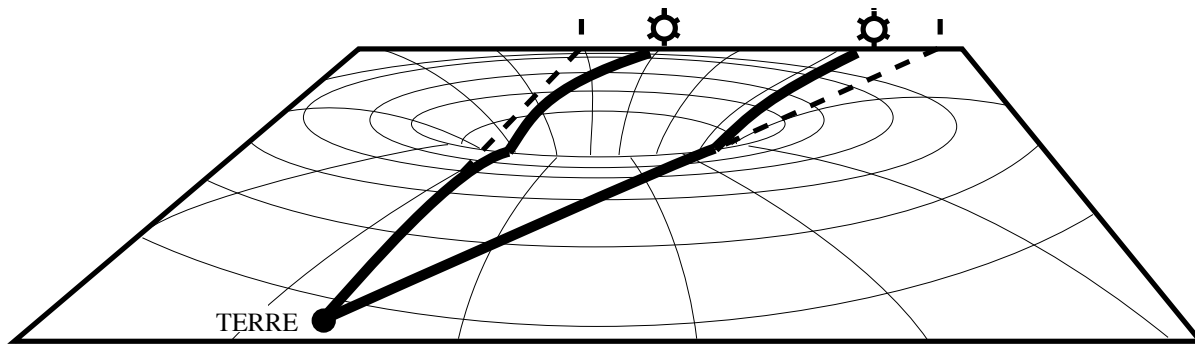
Déviatation de la lumière

Prédiction : la trajectoire de la lumière est modifiée par la présence d'une masse.

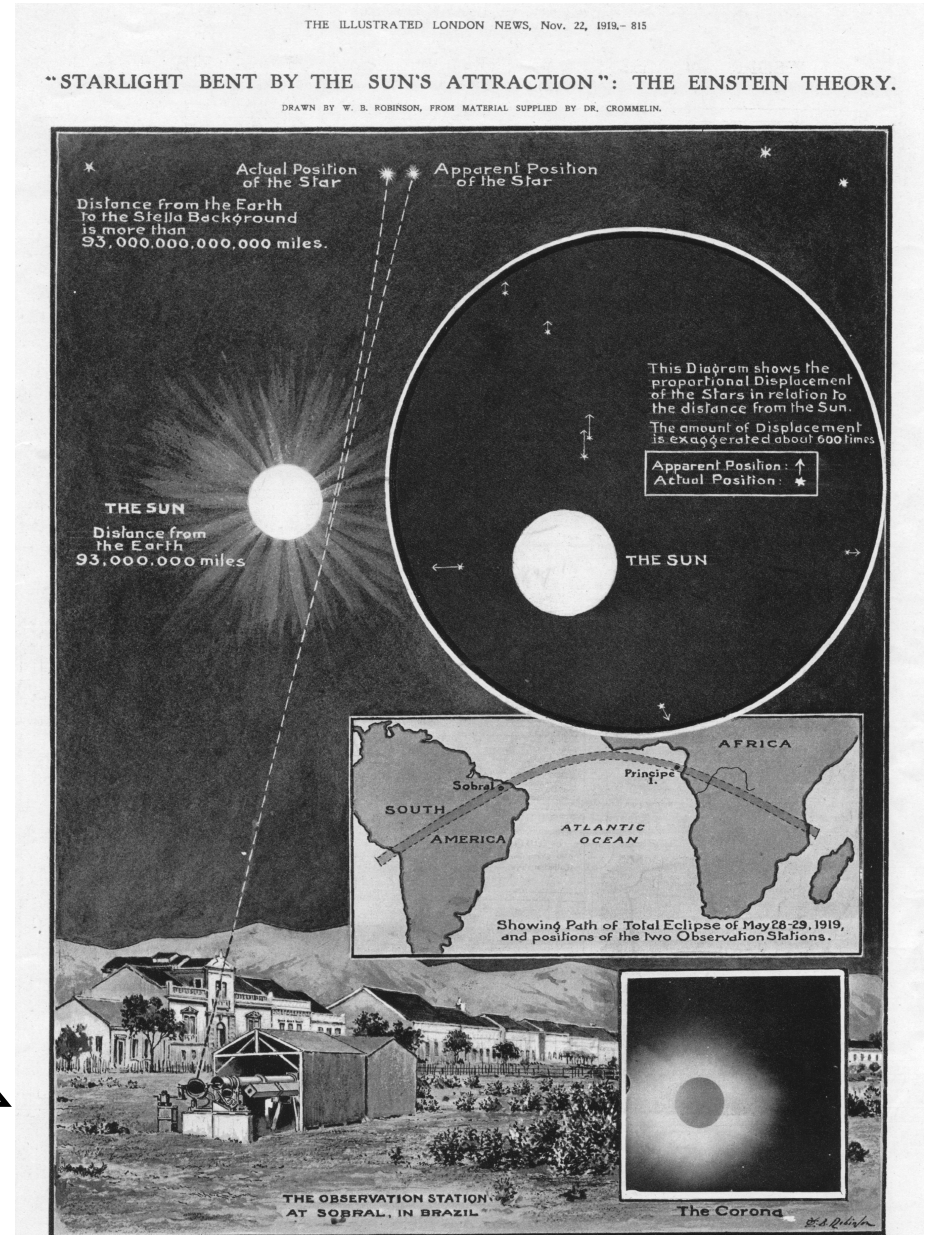
Angle de déviation $\alpha = \frac{4GM}{bc^2}$

Cas du Soleil $R_{\odot} = 696\,000\text{ km}$
 $\alpha \simeq 1.75''$

Pièce de 1 euro vue à 2,74 km



Couverture du *Illustrated London News*
 du 22 novembre 1919



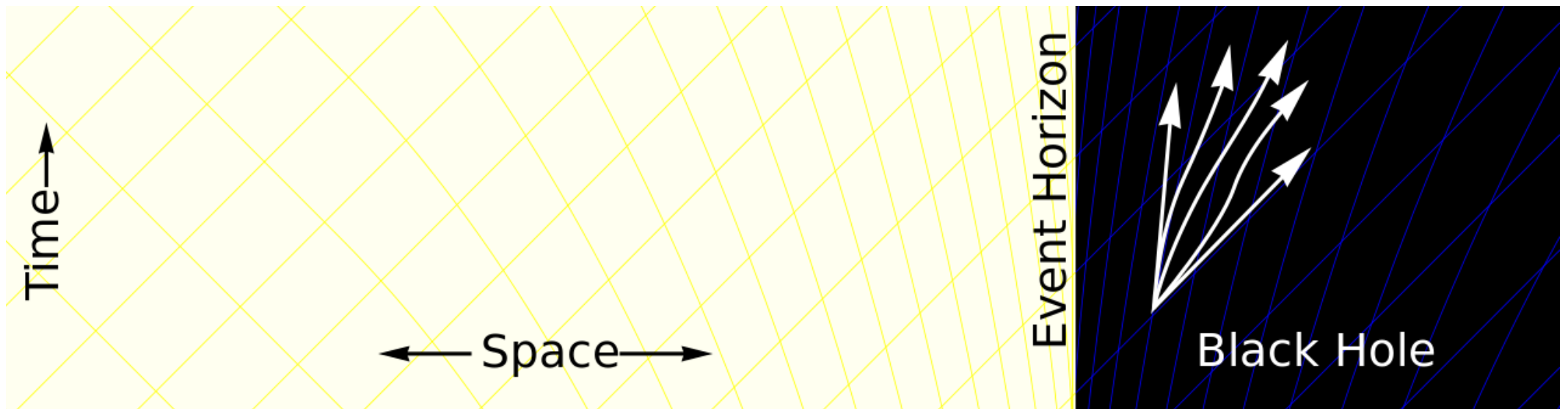
THE CURVATURE OF LIGHT: EVIDENCE FROM BRITISH OBSERVERS' PHOTOGRAPHS AT THE ECLIPSE OF THE SUN.

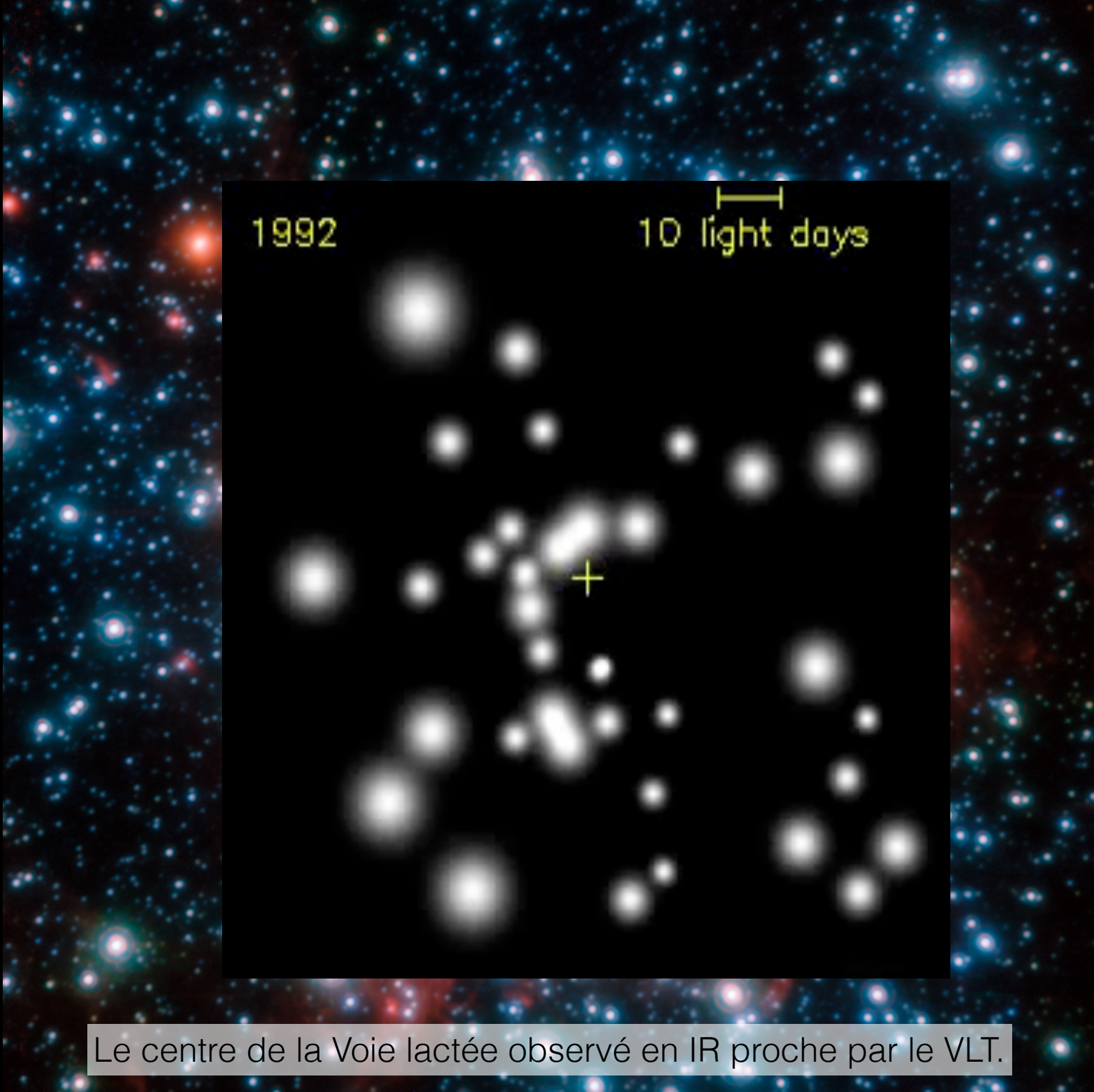
The results obtained by the British expeditions to observe the total eclipse of the sun last May verified Professor Einstein's theory that light is subject to gravitation. Writing in our issue of November 15, Dr. A. C. Crommelin, one of the British observers, said: "The eclipse was specially favourable for the purpose, there being no fewer than twelve fairly bright stars near the limb of the sun. The process of observation consisted in taking photographs of these stars during totality, and comparing them with other plates of the same region taken when the sun was not in the neighbourhood. Then if the starlight is bent by the sun's attraction, the stars on the eclipse plates would seem to be pushed outward compared with those on the other plates. . . . The second Sobral camera and the one used at Principe agree in supporting (Einstein's theory). . . . It is of profound philosophical interest. Straight lines in (Einstein's) space cannot exist; they are parts of gigantic curves." (Drawing Copyrighted in the United States and Canada.)

Qu'est-ce qu'un trou noir ?

C'est une région de l'espace courbée au point que la lumière ne peut s'échapper.

Rayon de Schwarschild $R_S = \frac{2GM}{c^2} \sim 3 \text{ km} \frac{M}{M_\odot}$





Le centre de la Voie lactée observé en IR proche par le VLT.

A composite image of the Centaurus A galaxy, showing its characteristic bar and lobes. The image is a combination of sub-mm, visible, and X-ray data. A red arrow points to the central region, labeled with the text 'Trou noir de 55 millions de masses solaires'. The galaxy is surrounded by a dense field of stars, with some bright stars showing prominent diffraction spikes.

Trou noir de 55 millions de masses solaires

Image composite – sub-mm, visible, X – de la galaxie Centaurus A (APEX/Chandra/ESO).



6 500 UA en 1 mois :
jet supraluminique !

La binaire X GRS 1915+105 (Aigle) abrite un trou noir de 14 masses solaires.

Peut-on voir un trou noir ?

Croix du Sud

α et β Cen

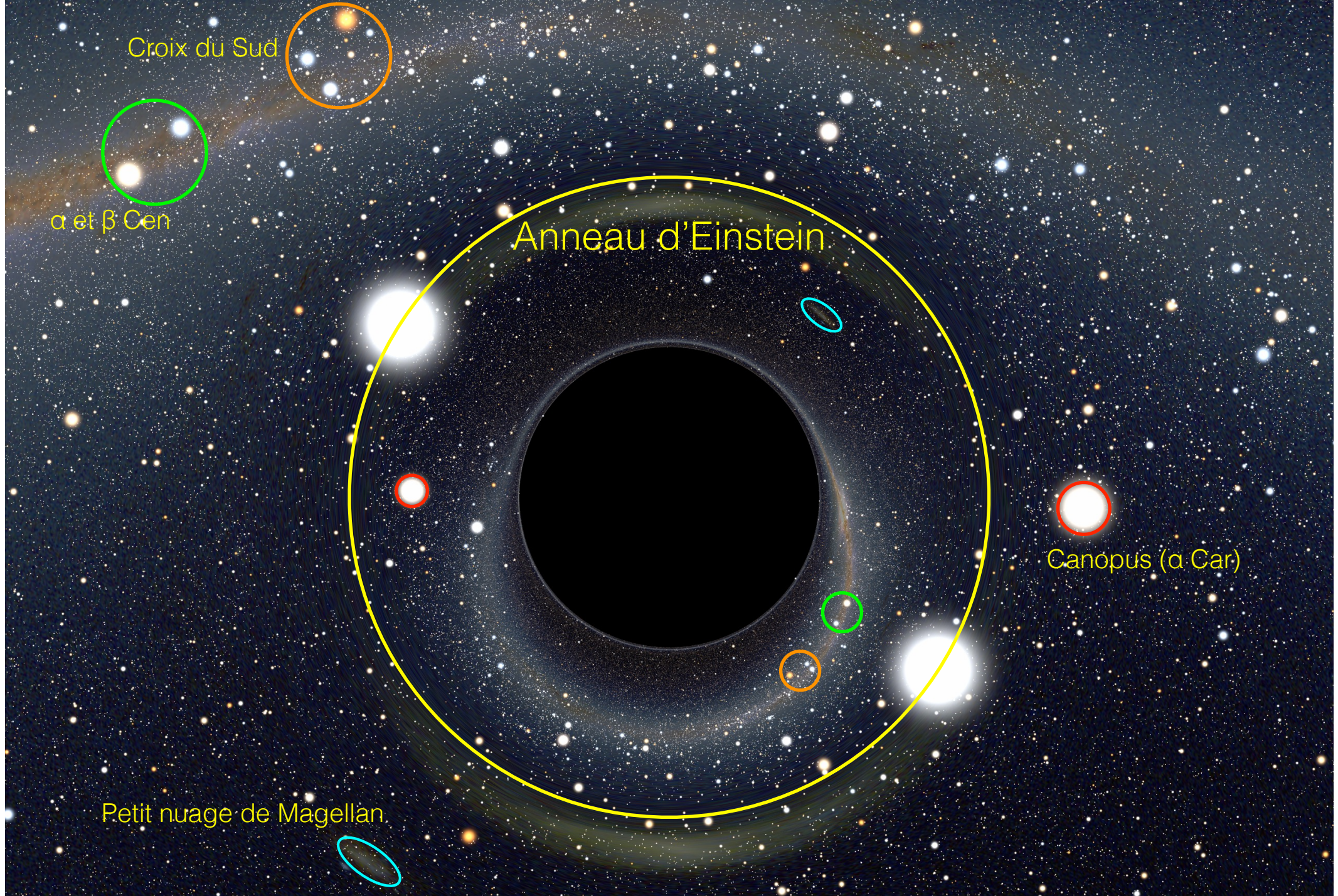
Sirius (α CMa)



Canopus (α Car)

Nuages de Magellan

Achernar (α Eri)



Croix du Sud

α et β Cen

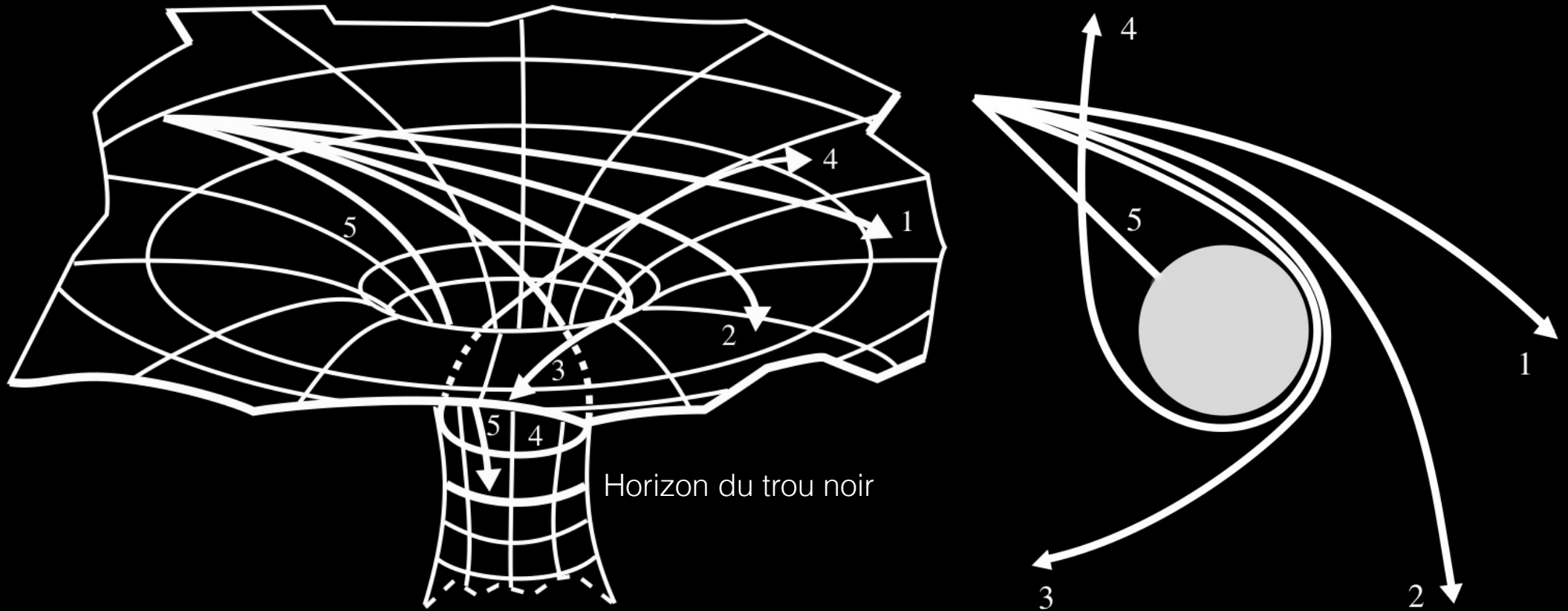
Anneau d'Einstein

Canopus (α Car)

Petit nuage de Magellan

Simulation d'A. Riazuelo (IAP/UPMC/CNRS, APOD 7/12/2010) ; $r = 10$
https://www.youtube.com/watch?v=A_KBd0kSIAc

Déviations de la lumière - 2



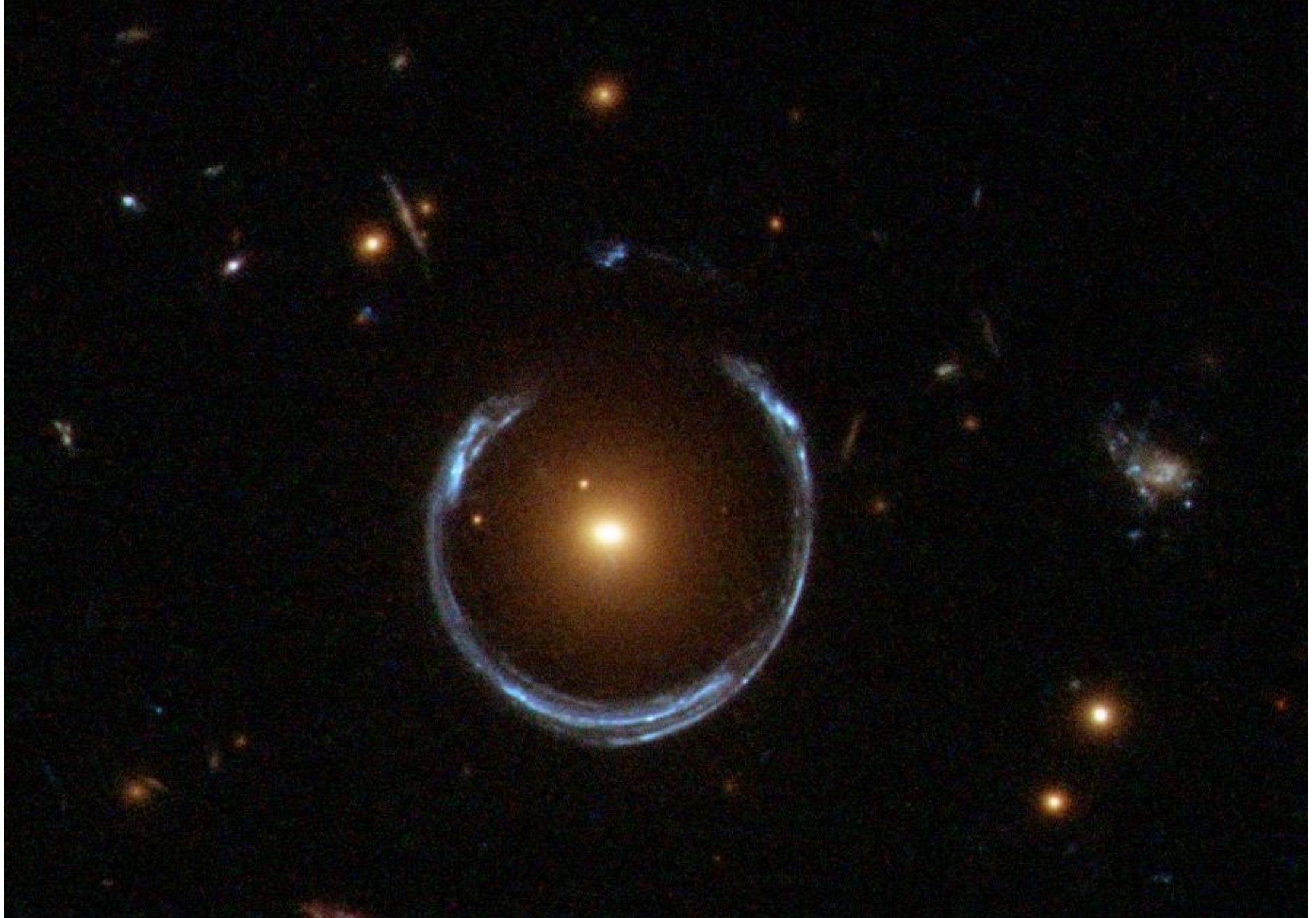
La lumière peut faire bien des tours autour d'un trou noir !

Images J.-P. Luminet

Lentille gravitationnelle

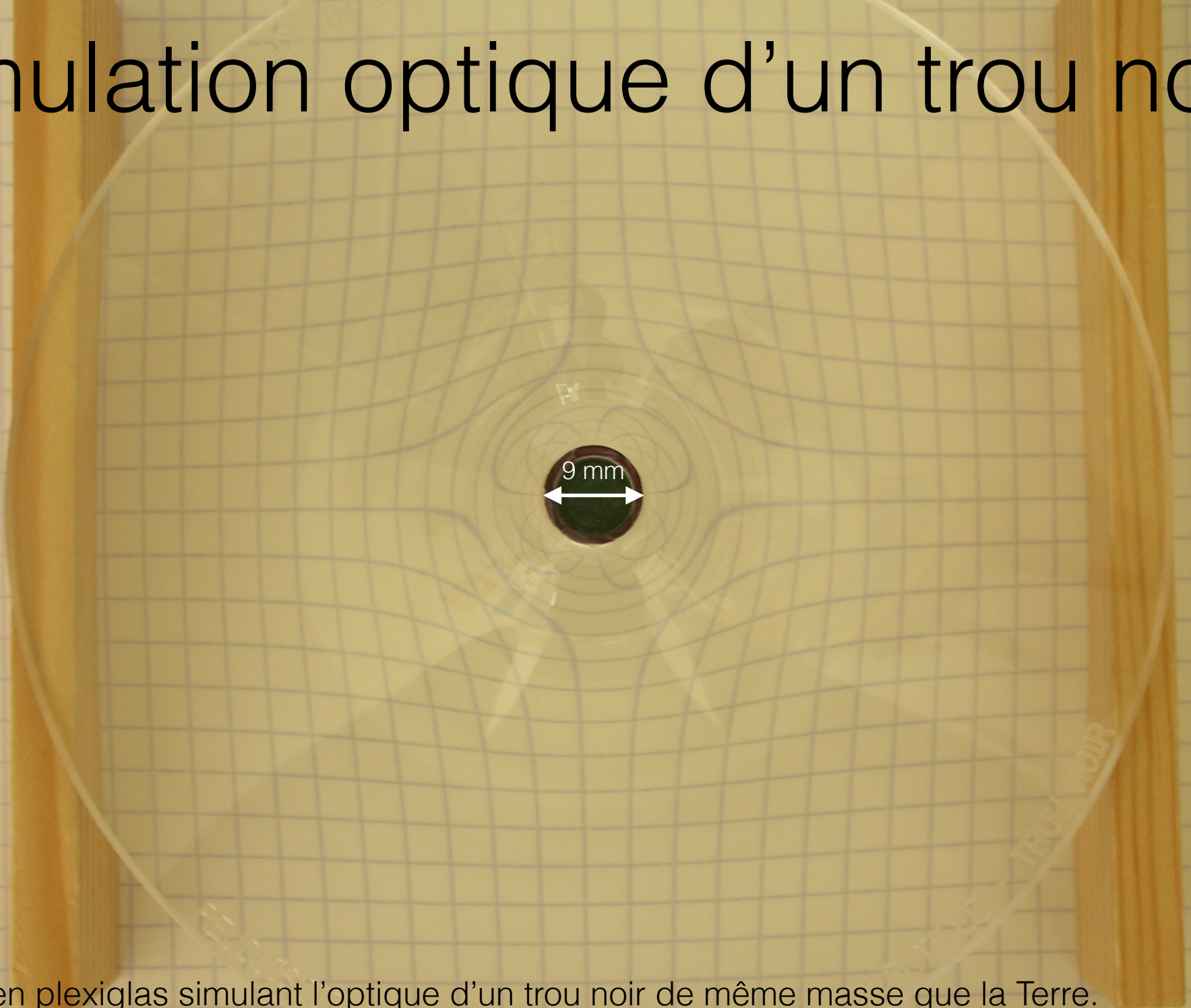


Effet de lentille gravitationnelle de l'amas de galaxies Abell 1689.



Anneau d'Einstein de la galaxie LRG 3-757 sur une galaxie bleue plus distante (HST).

Simulation optique d'un trou noir



Lentille en plexiglas simulant l'optique d'un trou noir de même masse que la Terre.
(Réjouissiences, Université de Liège)

T = 0.000 s
t = 0.000 s

v_loc = 56655 km/s
v_inf = 54734 km/s

En orbite autour d'un trou noir !

Effet du trou noir

distorsion, amplification et images multiples

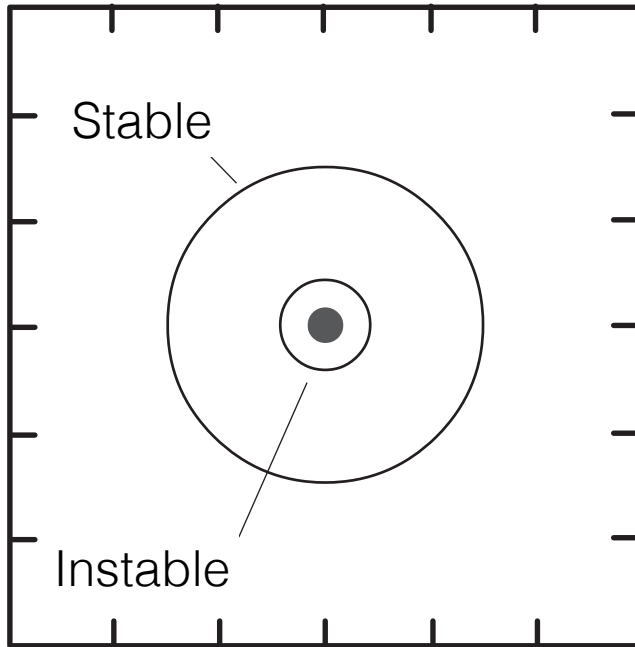
Simulation d'A. Riazuelo (IAP/UPMC/CNRS)

r = 30.000 M
 $\phi = 0.000^\circ$

z_min = -0.2021
z_grav = -0.0339
z_max = 0.1697

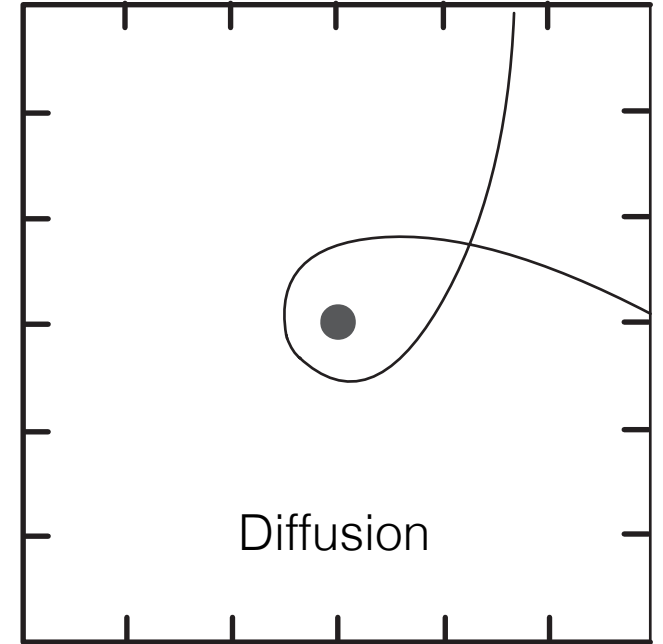


Orbites autour d'un trou noir

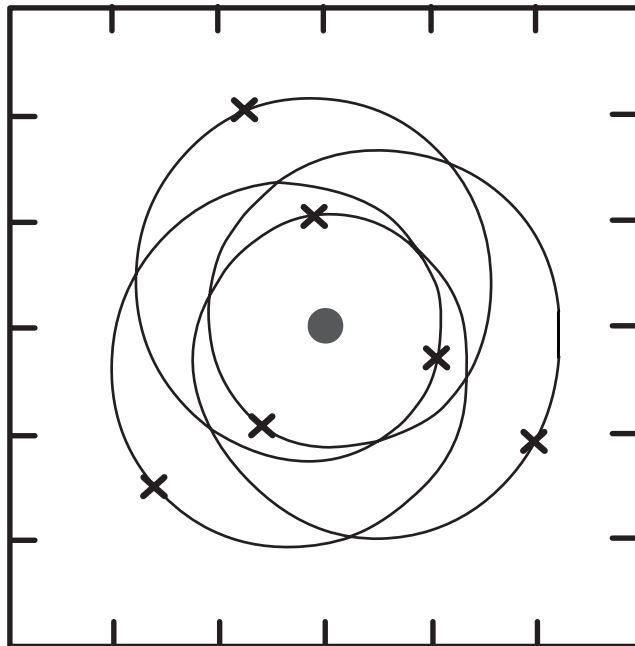


Orbites
circulaires

$$R_{\text{ISCO}} = \frac{6GM}{c^2} = 3R_S$$

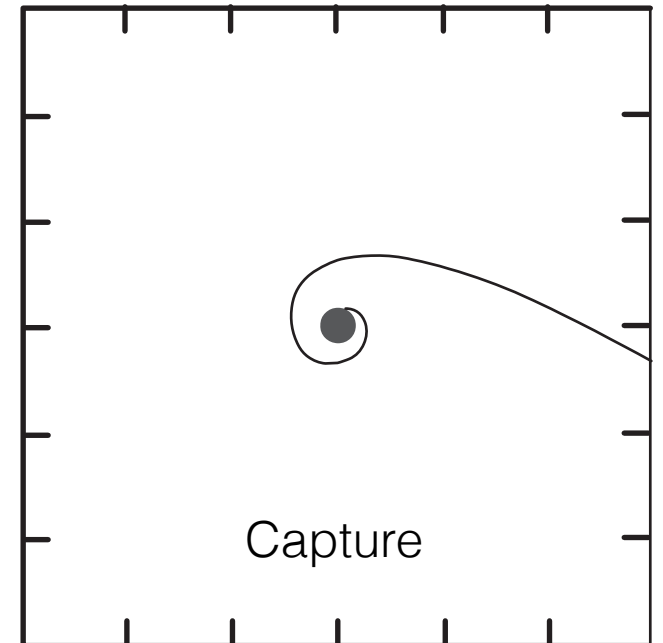


Diffusion



Orbites
fermées

Périhélie de Mercure : 43"/siècle.
Périastre du pulsar binaire : 4.2°/an.

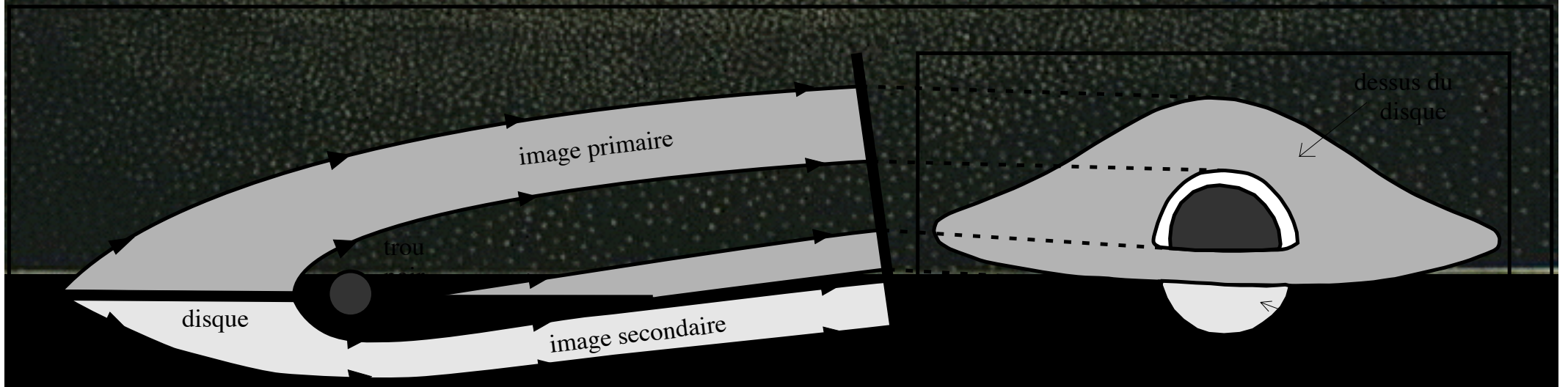
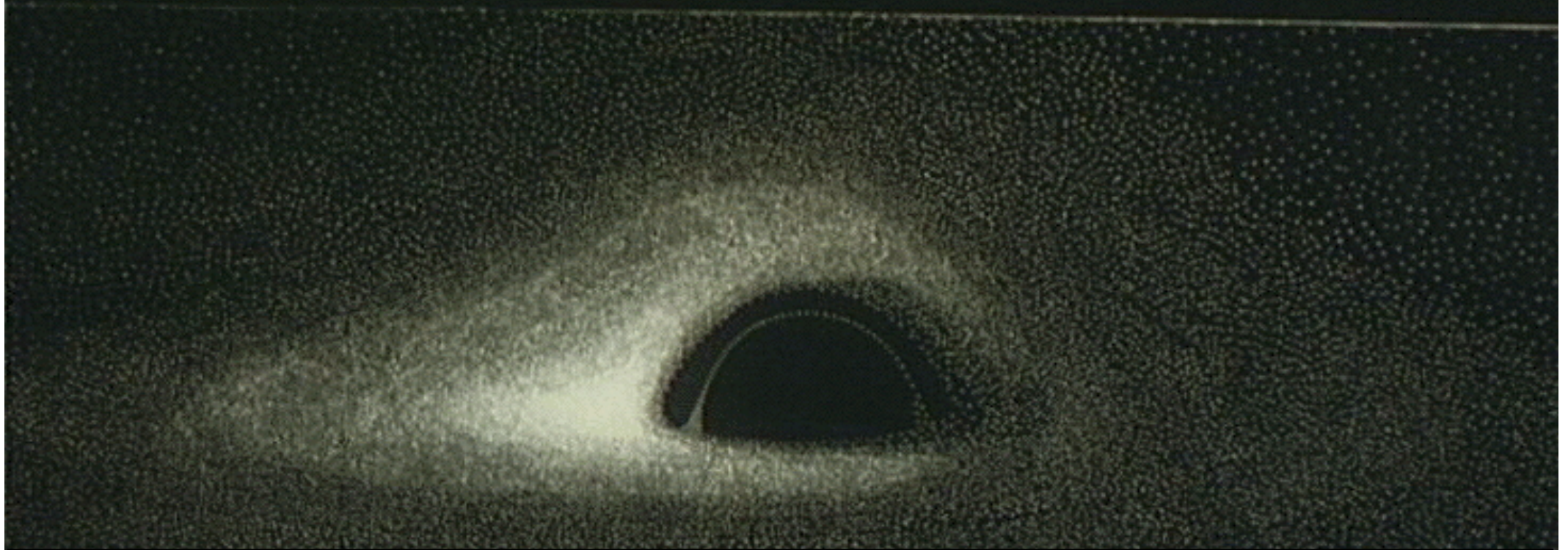


Capture

La lumière en orbite !

Les photons peuvent orbiter (instable) autour d'un trou noir statique à $R_{\gamma} = 3 GM/c^2 = 3/2 R_s$

Image D. Madore




Apparence d'un trou noir entouré d'un disque d'accrétion (J.-P. Luminet, 1979)



Apparence d'un trou noir entouré d'un disque d'accrétion (J.-A. Marck, 1995)

<https://www.youtube.com/watch?v=5Oqop50ltrM>



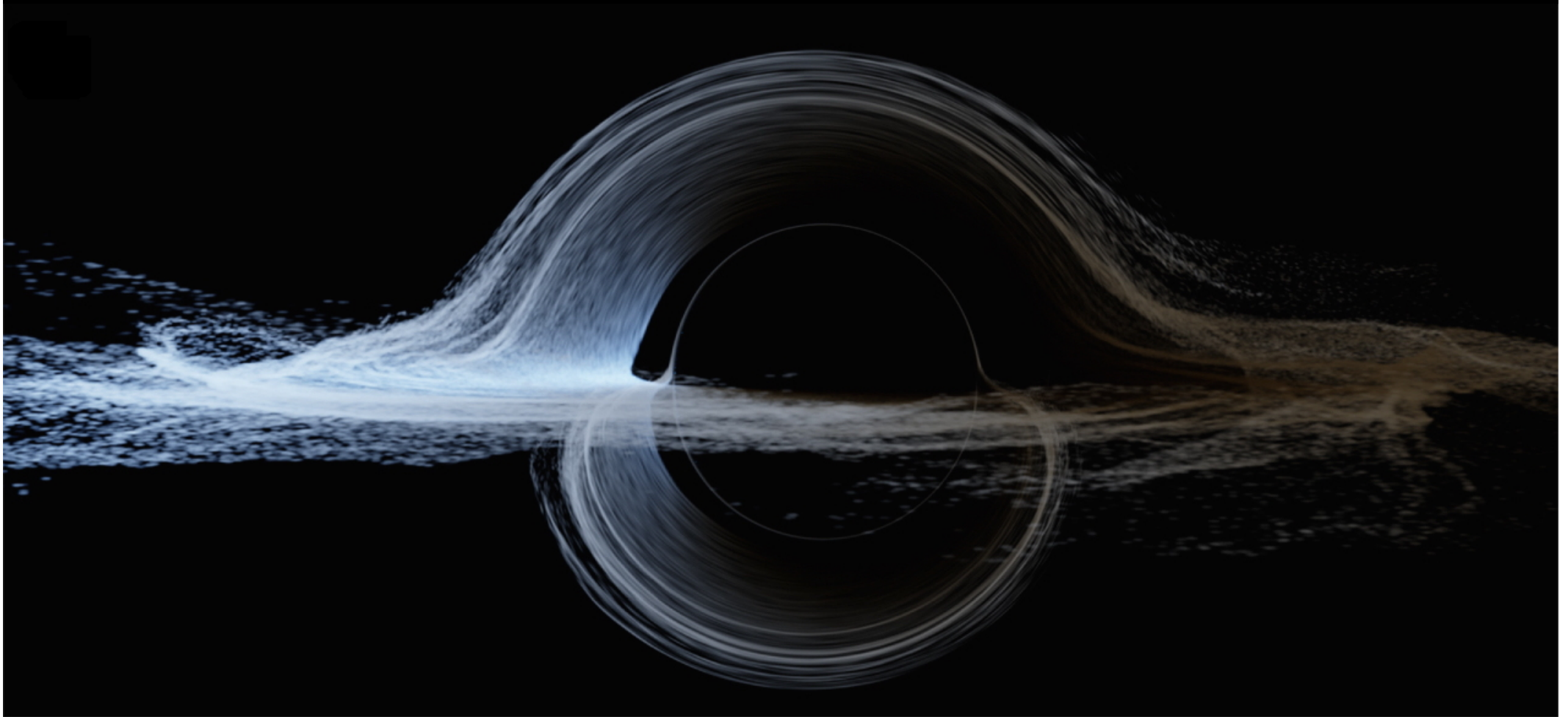
Partie supérieure
de l'arrière du disque

Le disque d'accrétion
qui brille en lumière visible...

Partie inférieure
de l'arrière du disque

Gargantua, son disque d'accrétion et la planète de Miller.

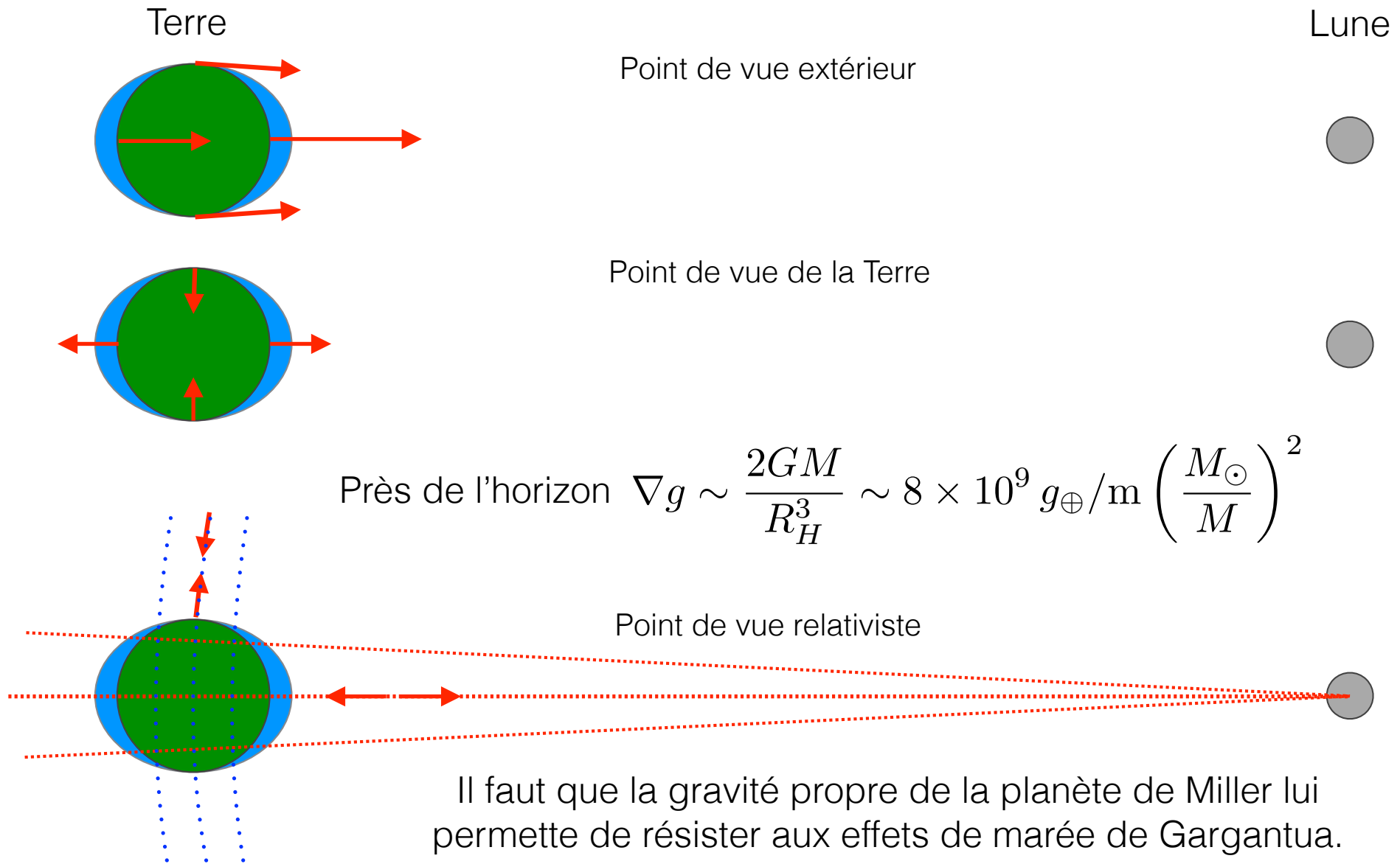
A Hollywood, même les trous noirs se maquillent !



Quelle est la masse
de Gargantua ?

Gargantua, son disque d'accrétion et la planète de Miller.

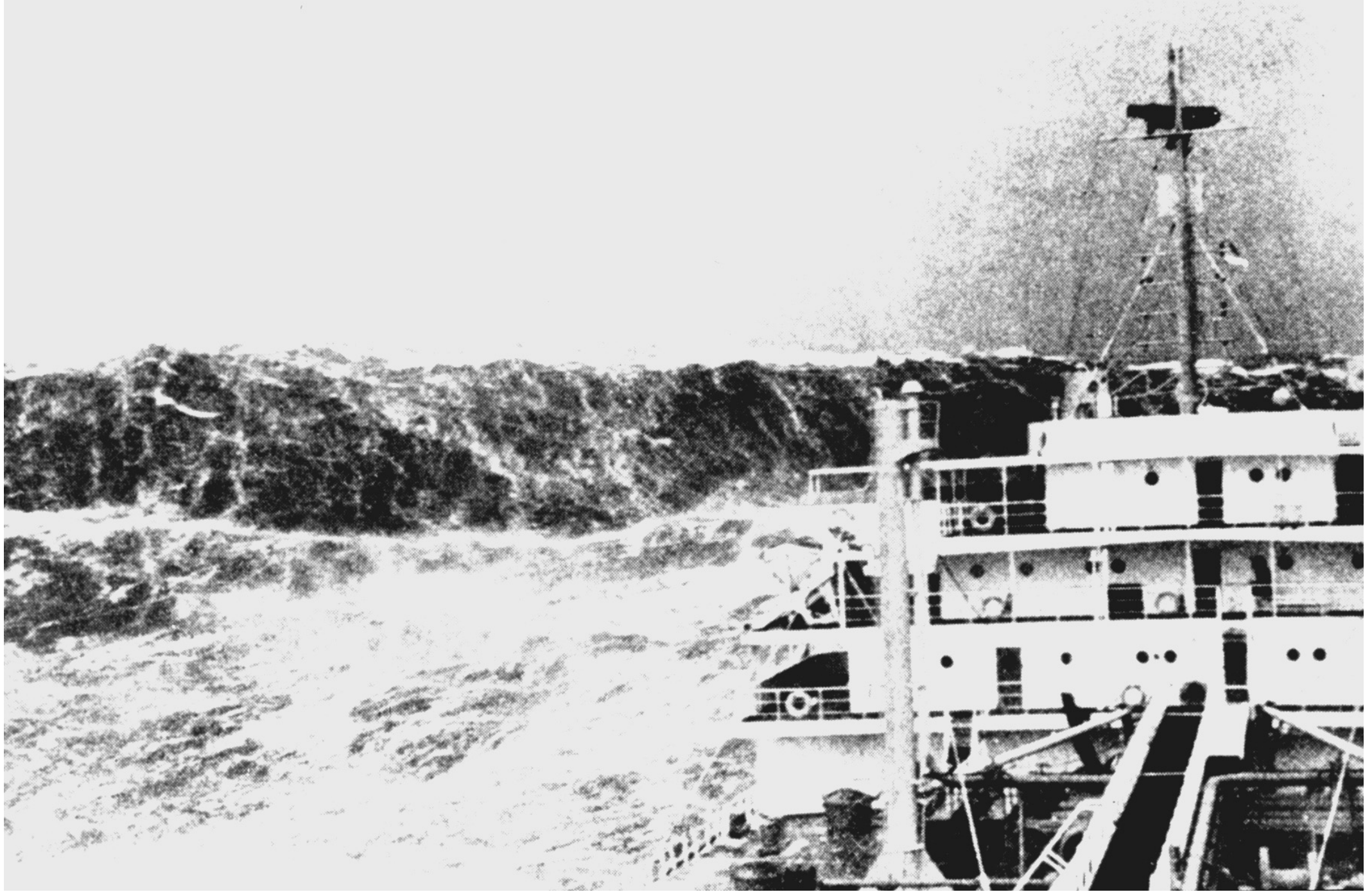
Effet de marée



La vague géante



La vague géante de la planète Miller.



Vague scélérat dans le Golfe de Gascogne, en 1940 (NOAA).



La grande vague de Kanagawa, Hokusai (1830), série « Trente-six vues du mont Fuji ».

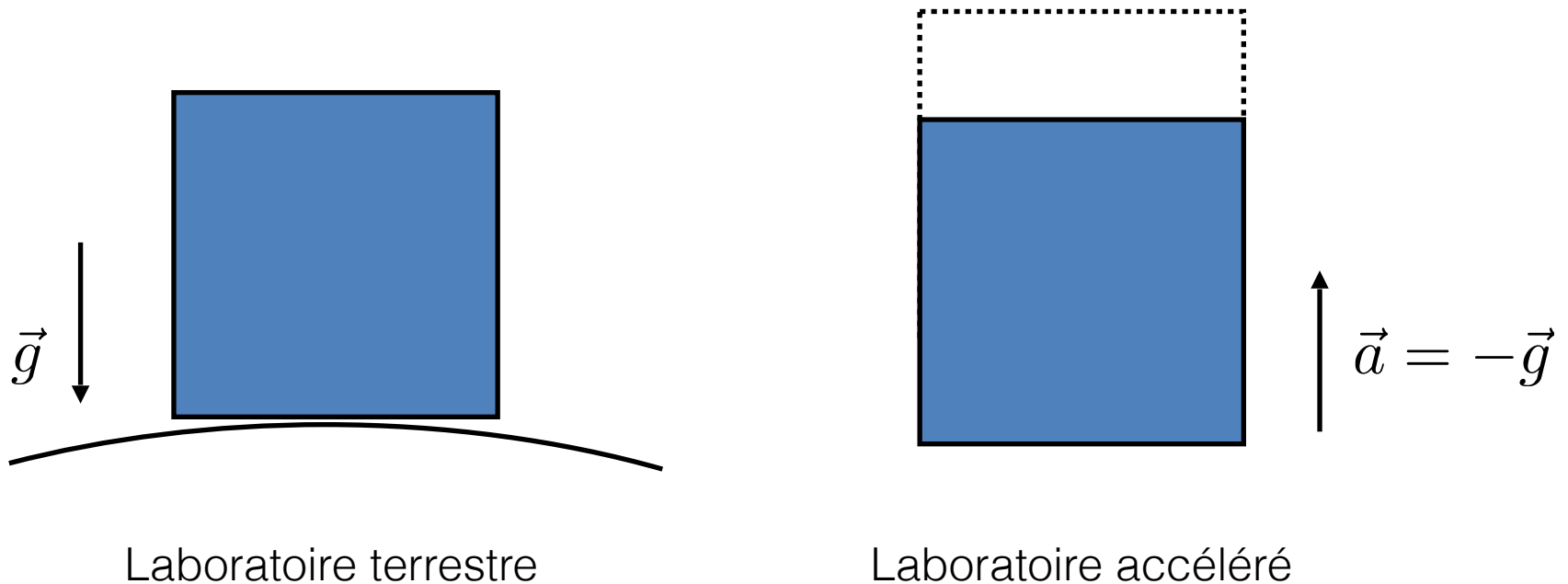
Le temps élastique

*Chaque heure passée sur cette planète équivaudra
à sept années sur notre bonne vieille Terre.*
Romilly

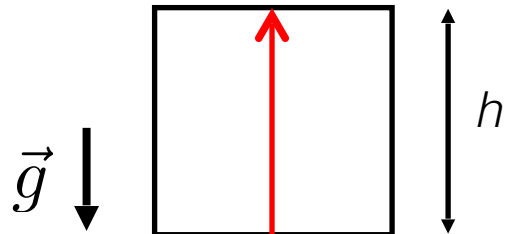
La descente du *Ranger* sur la planète de Miller.

Principe d'équivalence - le retour

Localement (dans l'espace et dans le temps),
les effets d'un **champ de gravité** sur une expérience n'utilisant pas la gravité
sont identiques à ceux d'une **accélération du référentiel** de l'observateur.

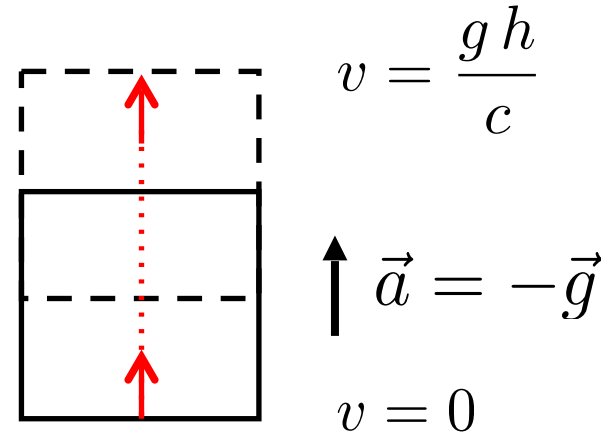


Redshift gravitationnel



Laboratoire terrestre

La lumière met une durée h/c pour atteindre le plafond.



Laboratoire accéléré

Le photon reçu est plus rouge (effet Doppler).

$$\nu_r \sim \nu_e \left(1 - \frac{v}{c}\right)$$

$$\nu_r \sim \nu_e \left(1 - \frac{gh}{c^2}\right) < \nu_e$$

Einstein (1907)

Vérifié par Pound & Rebka (1960)

Une horloge au sol retarde par rapport à une autre (identique) située en altitude.

Pour le GPS ce retard est de l'ordre de $46 \mu\text{s}$ par jour.

Cas de la planète Miller

Trou noir statique

$$\Delta\tau = \sqrt{1 - \frac{R_S}{r}} \Delta t$$

1 heure

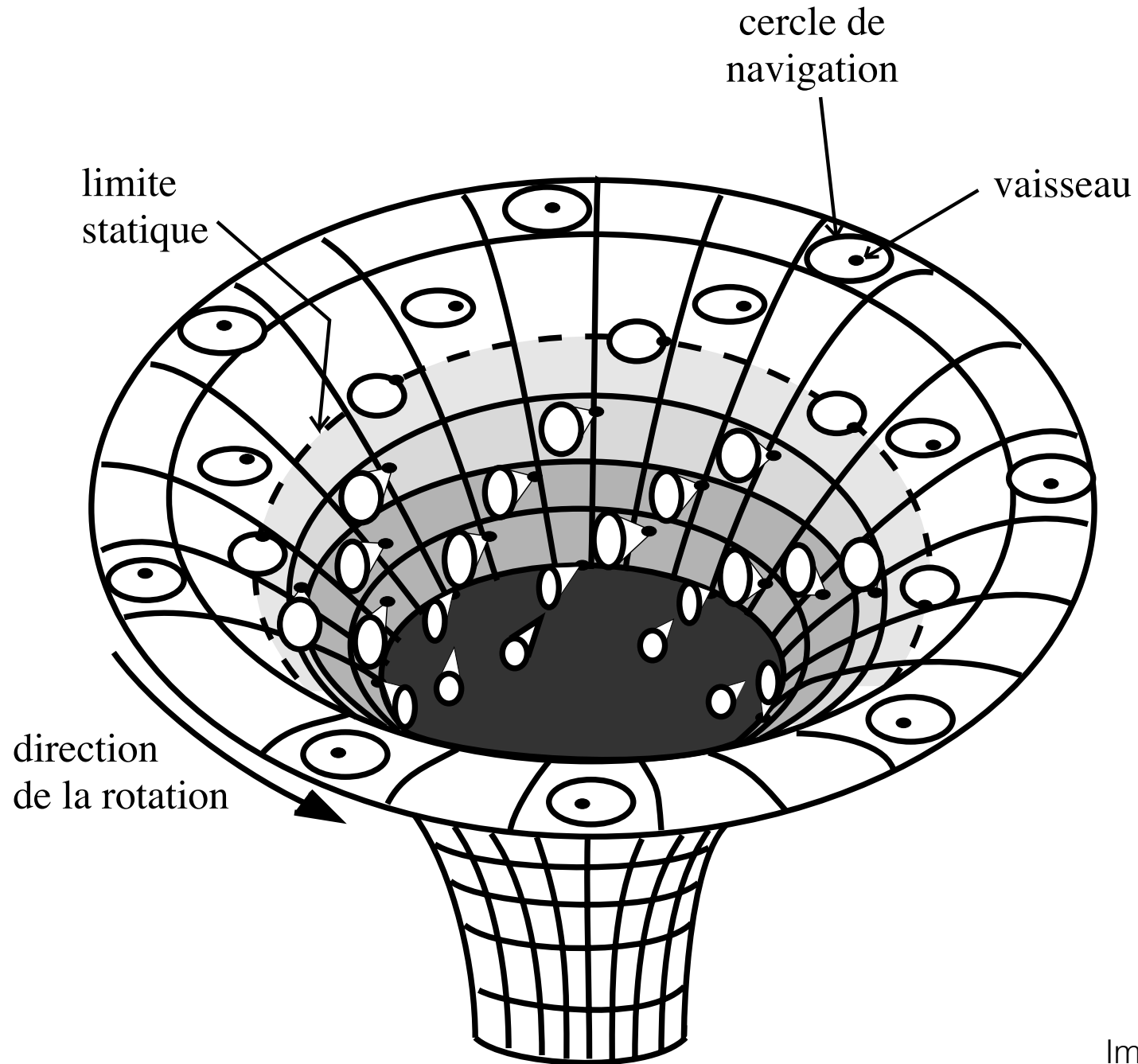
7 ans
61 320 heures

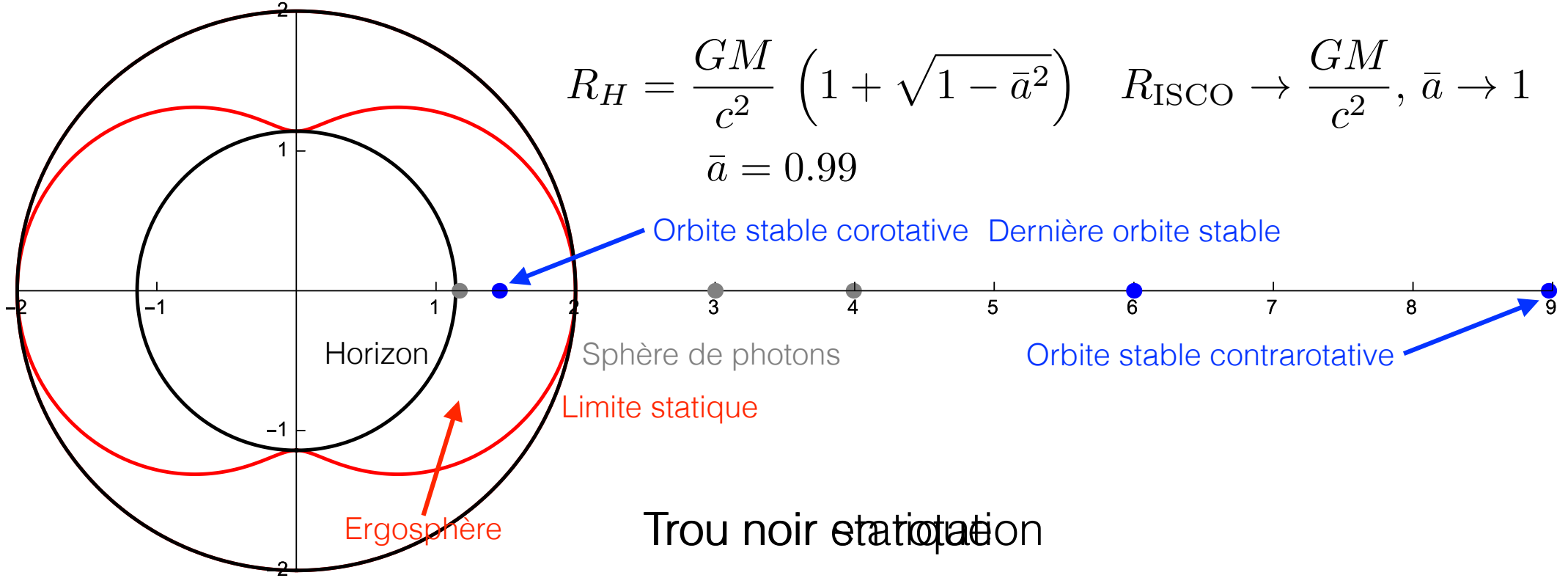
$$r = 1.000\,000\,000\,3 \times R_S$$

Mais cela signifie que la planète Miller est *très* proche de Gargantua...

Cette position est située à l'intérieur de la dernière orbite circulaire stable ($3 R_S$)...

Trou noir en rotation (de Kerr)



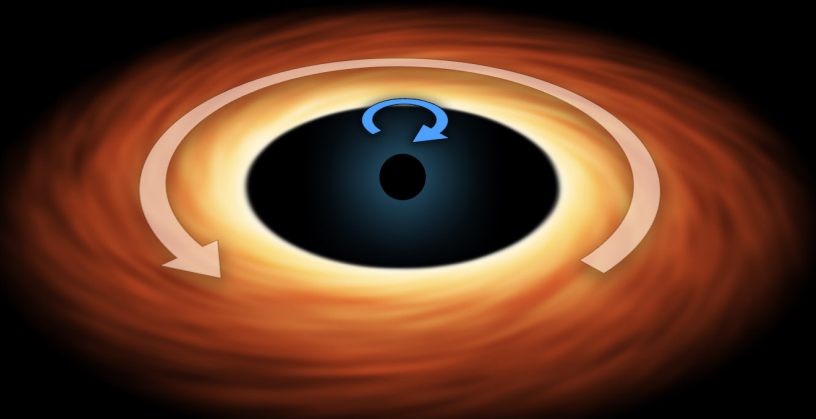


$$\frac{\Delta\tau}{\Delta t} \propto (1 - \bar{a})^{1/3}$$

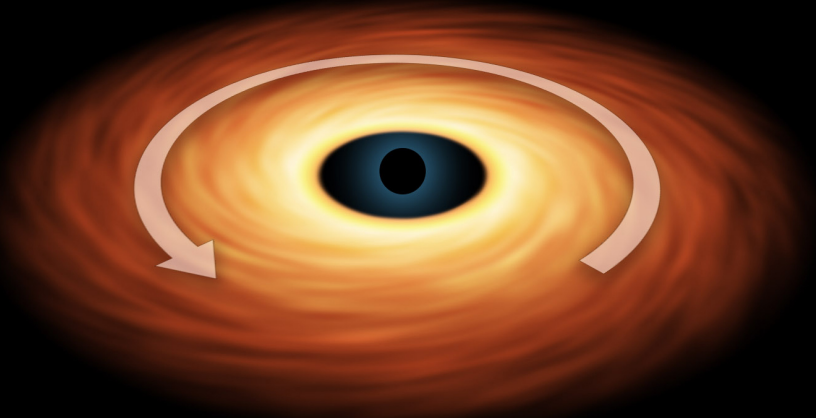
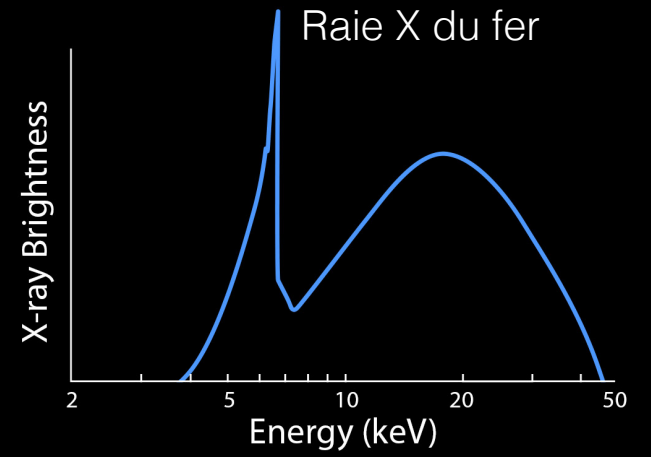
1 heure (pointing to $\Delta\tau$)
 7 ans
 61 320 heures (pointing to Δt)

$$1 - \bar{a} \approx 10^{-14}$$

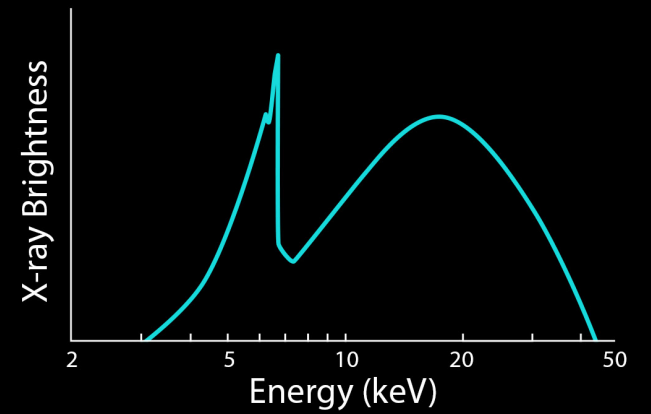
Gargantua est en rotation extrême !



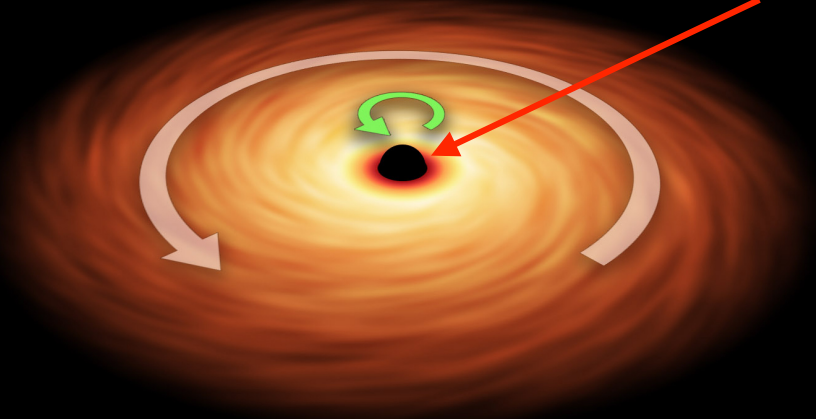
Retrograde
Rotation



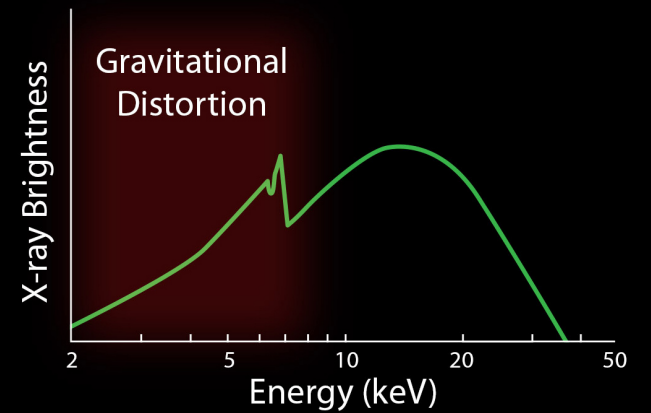
No Black Hole
Rotation



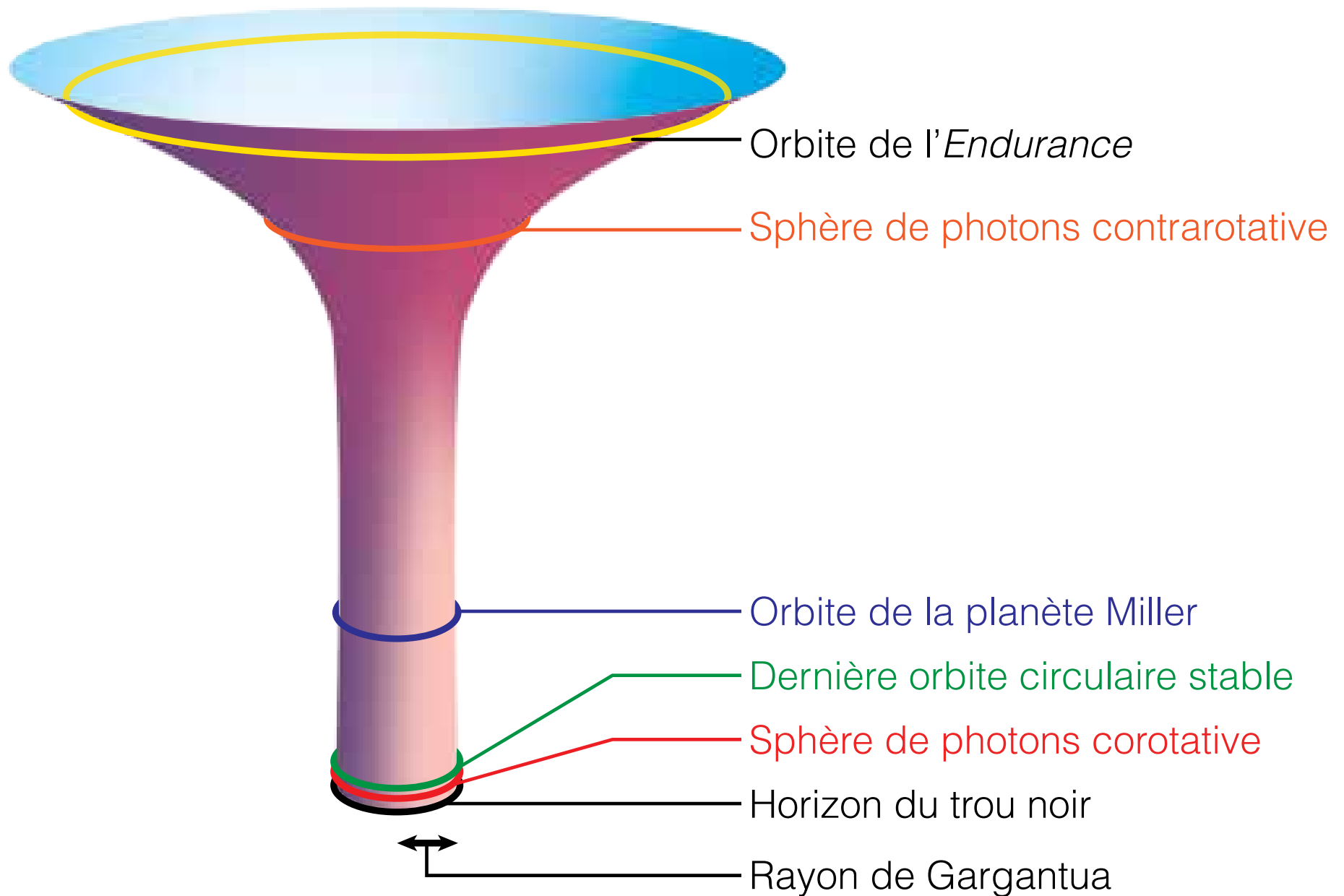
La planète Miller devrait être ici !



Prograde
Rotation



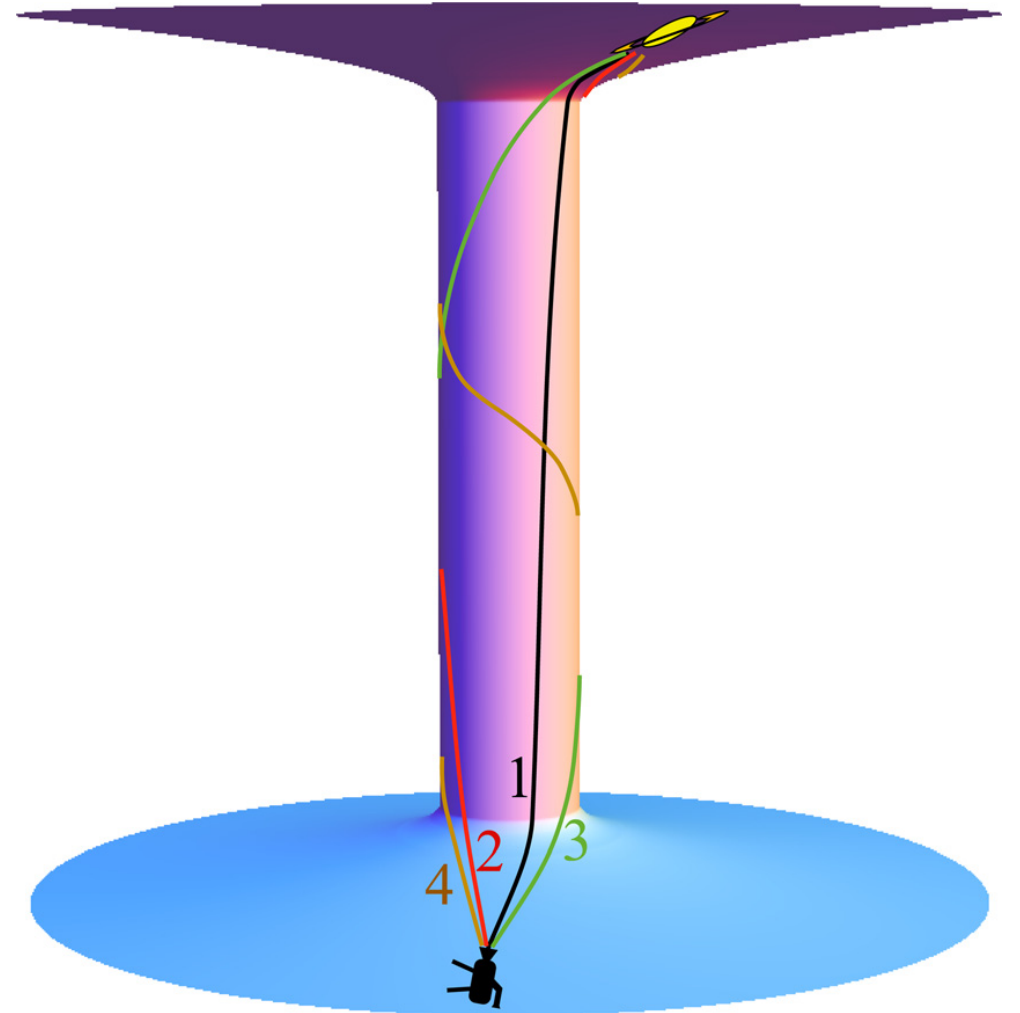
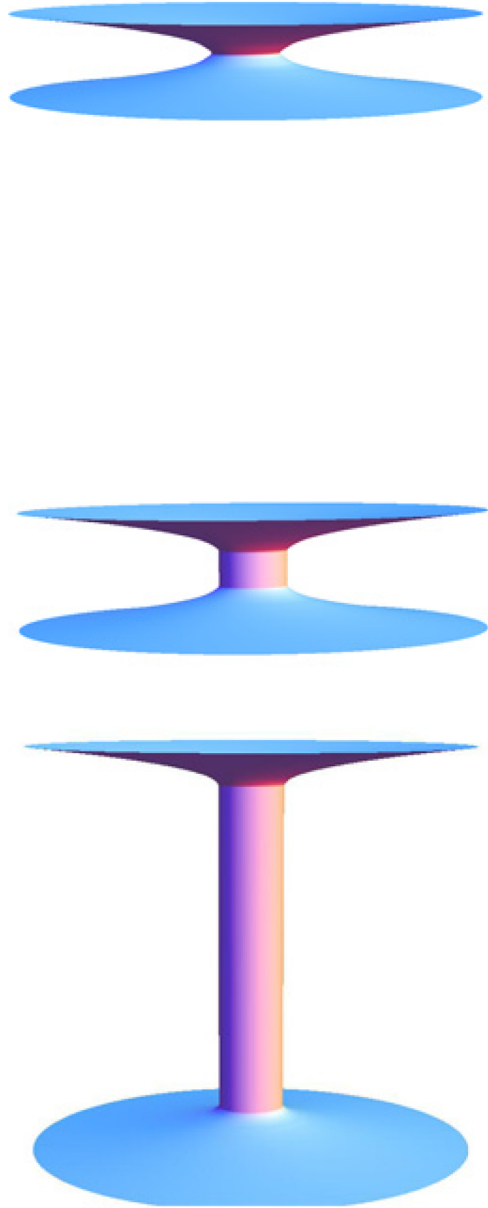
Bilan anatomique de Gargantua



Une porte sur l'univers



Le trou de ver situé près de Saturne



Images extraites de Am. J. Phys. 83, 486 (2015)

La station Cooper - 1

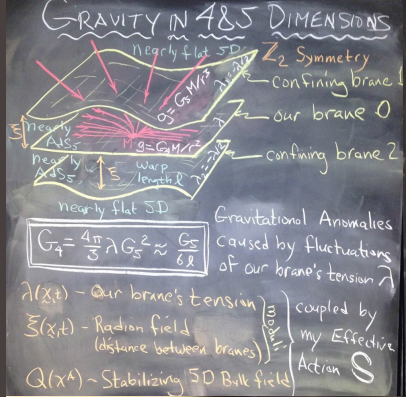


Vue intérieure de la station Cooper, en orbite autour de Saturne.

La station Cooper - 2



Joueurs de base-ball dans la station Cooper, en orbite autour de Saturne.



My Effective Action \$S(\lambda, \Xi, Q)\$ governs:

Variations \$\lambda(x,t) = \lambda(x,t)\$ of our brane's tension - and thence **ANOMALIES** \$G_4(x,t) = \frac{4\pi}{3} \lambda(x,t) G_5^2\$ in strength of gravity

Stability of the branes and thence of our Universe

All other features of our 4D & 5D Universe!

for the Effective Action \$S(\lambda, \Xi, Q)\$

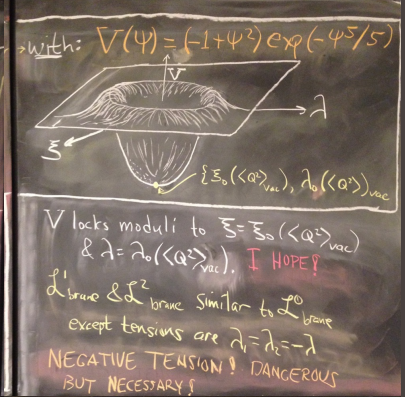
$$S = \int \sqrt{-g_5} d^5x \left\{ \mathcal{L}_{bulk} + \sum_{\text{brane location } a=0,2} \delta(\sigma(x^a) - \sigma_a) \mathcal{L}_{brane} \right\}$$

Where!

$$\mathcal{L}_{bulk} = \frac{1}{16\pi G_5} (R_5 - 2\Lambda_5) - \frac{1}{2} \nabla_a \Phi \nabla^a \Phi - \frac{1}{2} U(\Phi)$$

$$- \frac{1}{2} \sum_{\text{auxiliary fields } i,j} H_{ij}(\Phi) \nabla_a \Phi^i \nabla^a \Phi^j$$

$$\mathcal{L}_{brane} = \frac{K}{16\pi G_5} + \frac{1}{2} (Q^2 - 2) \nabla_\mu \Xi \nabla^\mu \Xi + \frac{1}{2} \nabla_\mu \lambda \nabla^\mu \lambda + \frac{1}{2} \sum_{i,j} W_{ij} \Phi^i \Phi^j \mathcal{M}(\text{standard model fields}) + \frac{1}{2} V(\sqrt{|\Xi - \Xi_0(\langle Q \rangle_{vac})|^2} + [\lambda - \lambda_0(\langle Q \rangle_{vac})]^2) + (\text{Standard Model Terms})$$



ISSUES TO SOLVE IN MY EQUATION FOR \$S\$

- Is \$Q\$ always in its vacuum quantum state - as I hope!
- How can I control \$\langle Q^2 \rangle_{vac}\$ - and thence our brane's tension \$\lambda_0(\langle Q^2 \rangle_{vac})\$ - and thence the Gravitational Anomalies \$G_4(x,t) = \frac{4\pi}{3} \lambda_0(\langle Q^2 \rangle_{vac}) G_5^2\$

Auxiliary Bulk Fields \$\{\Phi^i\}\$ may be the key!

I must iterate my equation \$S\$ for candidate sets of fields \$\{\Phi^i\}\$ & their \$H_{ij}(\Phi^2)\$, or \$W_{ij}, \mathcal{M}\$

- Compute evolutions of \$Q\$ and thence \$G_4\$
- See how standard model fields might drive \$\{\Phi^i\}, Q, G_4\$ without violating observations
- Compare with observed Anomalies
- Then iterate the set \$\{\Phi^i\}\$ again and again and again...

3. Is \$\langle Q^2 \rangle_{vac}\$ always \$> \lambda_0(\langle Q^2 \rangle_{vac})\$, so branes do not collide and destroy our universe?

Foundations for Solving my Equations Issues

- 5D Fundamental Constants: \$G_5\$ - Gravitational constant, \$\Lambda_5\$ - Cosmological constant
- Coordinates: In 5D spacetime: \$x^A\$, In 4D spacetime (our brane): \$x^\mu\$, Unit normal to our brane: \$n^A\$
- Fundamental fields in 5D - functions of \$x^A\$
- \$g_{AB}\$ - metric of spacetime, \$G_{AB}\$ - Einstein curvature tensor

\$Q\$ - stabilizing field
 \$\{\Phi^i\}\$ - set of auxiliary fields
 \$S_{TAB}\$ - stress-energy tensor

Fundamental fields on our 4D Brane - functions of \$x^\mu\$, i.e. \$(x,t)\$

\$\lambda\$ - brane's tension } moduli
 \$\Xi\$ - radion field }
 Standard-model fields
 \$g_{\mu\nu}\$ - metric of 4D spacetime
 \$G_{\mu\nu}\$ - Einstein curvature tensor

FOUNDATIONS - CONTINUED

5D Fundamental Equations

Einstein Equations

$$5G_{AB} = -\Lambda_5 g_{AB} + 8\pi G_5 S_{TAB}$$

Evolution of Non-gravitational fields

$$\delta \mathcal{L} / \delta Q = 0, \delta \mathcal{L} / \delta \Phi^i = 0, S_{TAB, 10} = 0$$

5D Junction Conditions Across Brane

\$g_{AB}\$ continuous

$$K_{\mu\nu}^+ - K_{\mu\nu}^- = -8\pi G_5 (T_{\mu\nu} - \frac{1}{3} T g_{\mu\nu})$$

jump in brane's extrinsic curvature

$$S_{TAB}^+ - S_{TAB}^- = -T_{\mu\nu}^{\mu\nu} + \lambda \delta_{\mu\nu}$$

4D Fundamental Equations (on our brane)

4D Einstein Equations

$$G_{\mu\nu} = -\Lambda_4 g_{\mu\nu} + 8\pi G_4 T_{\mu\nu} + (8\pi G_5)^2 S_{\mu\nu}$$

\$-E_{\mu\nu} + \frac{2}{3} (8\pi G_5)^2 T_{\mu\nu}\$

Hold Wormhole open!

Here:

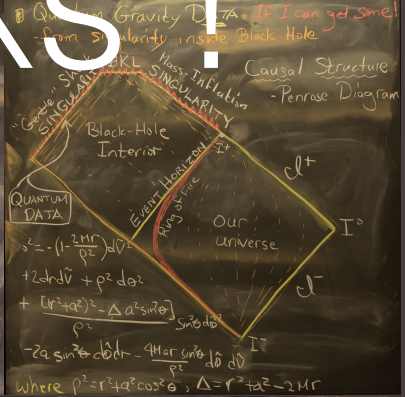
$$S_{\mu\nu} = \frac{1}{2} T_{\mu\nu} - \frac{1}{4} T g_{\mu\nu} + \frac{2}{3} g_{\mu\nu} (S_{TAB}^{\mu\nu} - T)$$

- generally very small

Evolution of brane tension \$\lambda(x,t)\$ and radion field \$\Xi(x,t)\$

$$\frac{\delta \mathcal{L}_{brane}^0}{\delta \lambda} = 0, \frac{\delta \mathcal{L}_{brane}^0}{\delta \Xi} = 0$$

Energy-Momentum Conservation

$$T_{\alpha\beta};\beta = \lambda_{;\alpha} - [S_{TAB}^{\mu\nu} n^{\mu\nu} - S_{TAB}^{\mu\nu} n^{\mu\nu}]$$


That's all folks!

SOLUTION OF 5D EINSTEIN EQUATIONS

5D Stress-Energy Tensor: from my \$S\$

$$S_{TAB} = \sum_{i,j} H_{ij}(\Phi) \nabla_a \Phi^i \nabla^a \Phi^j - \frac{1}{2} g_{AB} \mathcal{L}_{brane} + \delta \Phi^i \nabla^a \Phi^j \nabla_a \Phi^k$$

Solution of \$5G_{AB} = -\Lambda_5 g_{AB} + 8\pi G_5 S_{TAB}\$ - perturbations are **MUST DOMINATE!!**

Set \$\Lambda_5 = -6/R^2\$

Then if \$\Lambda_5\$ dominates: We live in a Gregory-Rubakov-Sibiryakov type universe (variant of Randall Sundrum):

Between branes: Almost AdS5

$$S_{AB} = e^{-2M/r} \rho_{AB}^2 dx^\mu dx^\nu + \rho_{AB}^2 (dx^5)^2 + (\text{perturbations})$$

- our brane @ \$U=0\$; companions @ \$U=\pm \frac{1}{2} R(x,t)\$

Outside Confining Branes: Almost Flat

$$S_{AB} = (5D \text{ Minkowski}) + (\text{perturbations})$$

4D Gravity and Brane Tensions

$$G_4 = \frac{4\pi}{3} \lambda_0(\langle Q^2 \rangle_{vac}) G_5^2 \approx G_5 / l$$

$$\lambda = \lambda_0(\langle Q^2 \rangle_{vac}) \approx \frac{3}{4\pi G_5 l}$$

$$\lambda_1 = \lambda_2 = -\frac{1}{2} \lambda_0(\langle Q^2 \rangle_{vac}) \approx -\frac{3}{8\pi G_5 l}$$

$$\Lambda_4 = \frac{1}{2} (\Lambda_5 + 8\pi G_4 \lambda) \approx 0$$

When anomalies are small

EVOLUTION EQUATIONS

Controlling \$Q\$, & thence \$\lambda\$, & thence \$G_4\$ & Anomalies

Evolution of Auxiliary Fields \$\{\Phi^i\}\$: observe \$\delta \Phi^i = 0\$

$$\nabla^a [\sum_{i,j} H_{ij}(\Phi) \nabla_a \Phi^i] = \delta(\sigma(x^a) - \sigma_a) \sum_{i,j} W_{ij}(\Phi) \mathcal{M}(\text{SM fields})$$

delta function on our brane

Evolution of Stabilizing Field, \$Q\$: \$\delta \mathcal{L} / \delta Q = 0\$

$$\nabla^a \nabla_a Q - U'(Q) = Q \nabla_\mu \Xi \nabla^\mu \Xi - \delta(\sigma(x^a) - \sigma_a) + (\text{Standard Model Fields})$$

Auxiliary field \$\{\Phi^i\}\$ drive \$Q\$, thence \$\langle Q^2 \rangle_{vac}\$

Evolution of our Brane's Tension: \$\delta \mathcal{L}_{brane}^0 / \delta \lambda = 0\$

$$\nabla^a \nabla_a \lambda = -\frac{1}{2} \nabla_\mu \Xi \nabla^\mu \Xi + [\lambda - \lambda_0(\langle Q \rangle_{vac})] V(\sqrt{|\Xi - \Xi_0(\langle Q \rangle_{vac})|^2} + [\lambda - \lambda_0(\langle Q \rangle_{vac})]^2)$$

\$= 0\$ when \$\Xi = \Xi_0(\langle Q \rangle_{vac})\$ & \$\lambda = \lambda_0(\langle Q \rangle_{vac})\$ - minimum of \$V\$

I hope \$\lambda\$ gets locked onto \$\lambda_0(\langle Q^2 \rangle_{vac})\$ - if not! **TROUBLE!**

Evolution of Radion Field: \$\delta \mathcal{L}_{brane}^0 / \delta \Xi = 0\$

$$(\langle Q^2 \rangle_{vac}) \nabla^a \nabla_a \Xi = [\Xi - \Xi_0(\langle Q \rangle_{vac})] V(\sqrt{|\Xi - \Xi_0(\langle Q \rangle_{vac})|^2} + [\lambda - \lambda_0(\langle Q \rangle_{vac})]^2)$$

where \$\lambda = \lambda_0(\langle Q \rangle_{vac})\$ & \$\Xi = \Xi_0(\langle Q \rangle_{vac})\$ - minimum of \$V(\Phi)\$

which I hope locks \$\lambda\$ onto \$\lambda_0(\langle Q^2 \rangle_{vac})\$ and keeps \$\langle Q^2 \rangle_{vac} > \lambda_0(\langle Q^2 \rangle_{vac})\$

THEREBY: preventing branes from colliding and thence saving our universe from destruction (in accord with observation?)

Bibliographie

1. Voyage autour et à l'intérieur d'un trou noir, par A. Riazuelo : <https://www.youtube.com/watch?v=uSrIIRaljKg>
2. Le documentaire *Infiniment courbe* (52 minutes), de L. Delesalle, M. Lachièze-Rey et J.-P. Luminet
<https://www.youtube.com/watch?v=wOVRXd-4jrQ>
<https://www.youtube.com/watch?v=Ilt8ioLPtEY>
3. The Science of Interstellar, K. Thorne, Norton Company, 2014.
4. Trous noirs et distorsions du temps, K. Thorne, Flammarion collection Champs, 2009.
5. Le destin de l'univers, J.-P. Luminet, Folio/Gallimard, 2010.
6. Plongeon dans un trou noir, J.-A. Marck et J.-P. Luminet, Pour la Science HS juillet 1997 p. 50-56.
7. Article scientifique de K. Thorne et de son équipe de simulation d'images
Classical and Quantum Gravity, 32, 065001 (2015) ; disponible sur <http://iopscience.iop.org/0264-9381/32/6/065001/article>